Name: _____ **Section Number:** _____

TA Name: _____ **Section Time:** _____

Math 20B.
Final Examination
March 17, 2004

You may use one page of notes, but no other assistance on this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

1. (4 points) Use the comparison test to determine whether the improper integral

$$\int_0^\infty \frac{\sin^2(x)}{e^x}\,dx$$

converges or diverges.

#	Score
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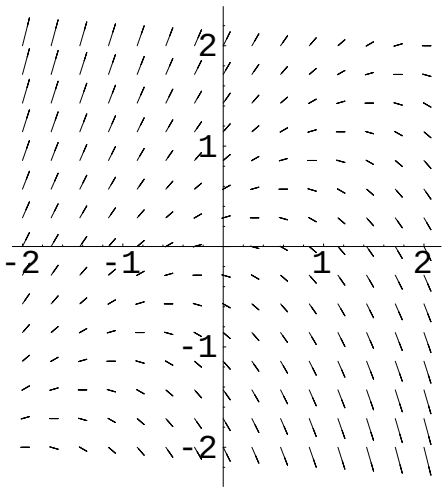
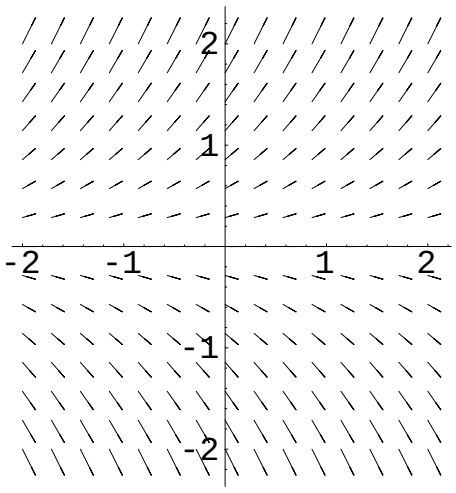
2. (5 points) Clearly circle the corresponding direction field for each of the following differential equations. Only one answer will be accepted per question. (Yes, there is an extra direction field.)

(i)	$y' = x$	A	B	C	D	E	F
(ii)	$y' = y$	A	B	C	D	E	F
(iii)	$y' = x + y$	A	B	C	D	E	F
(iv)	$y' = 1 - y^2$	A	B	C	D	E	F
(v)	$y' = y - x$	A	B	C	D	E	F

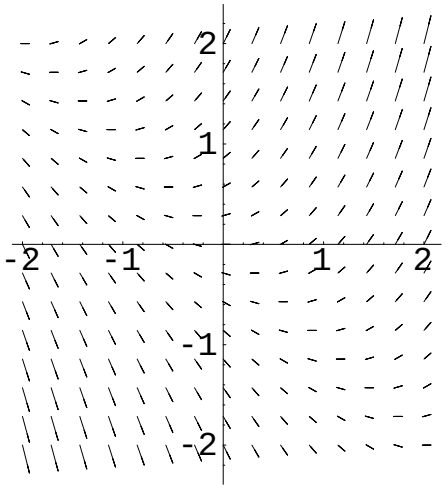
(A)



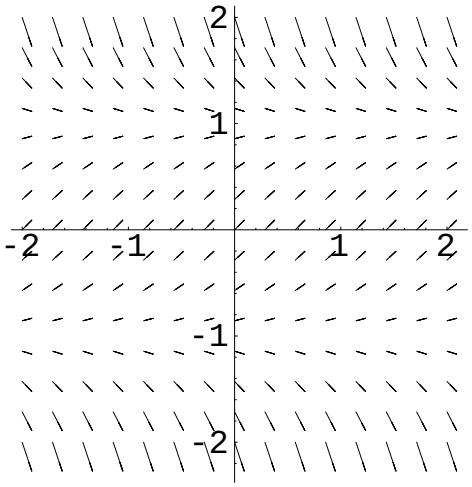
(B)



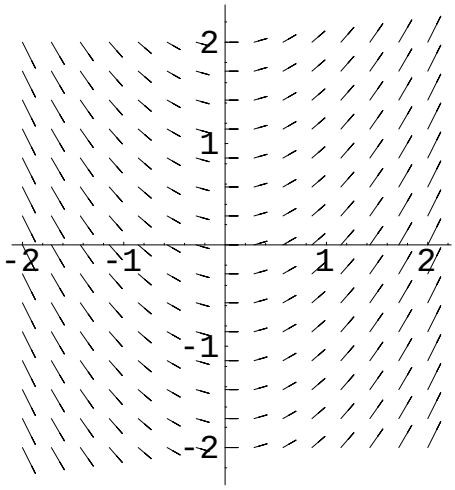
(C)



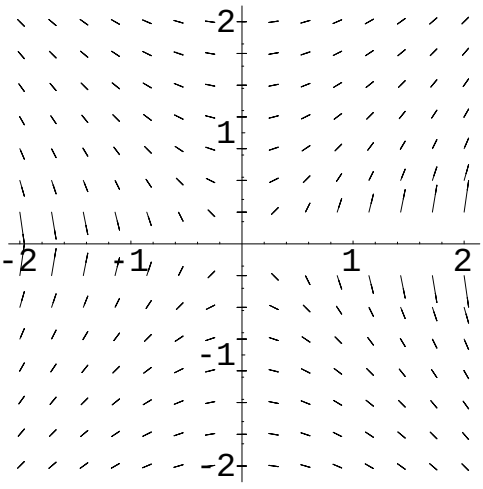
(D)



(E)



(F)



3. (4 points) Evaluate the indefinite integral

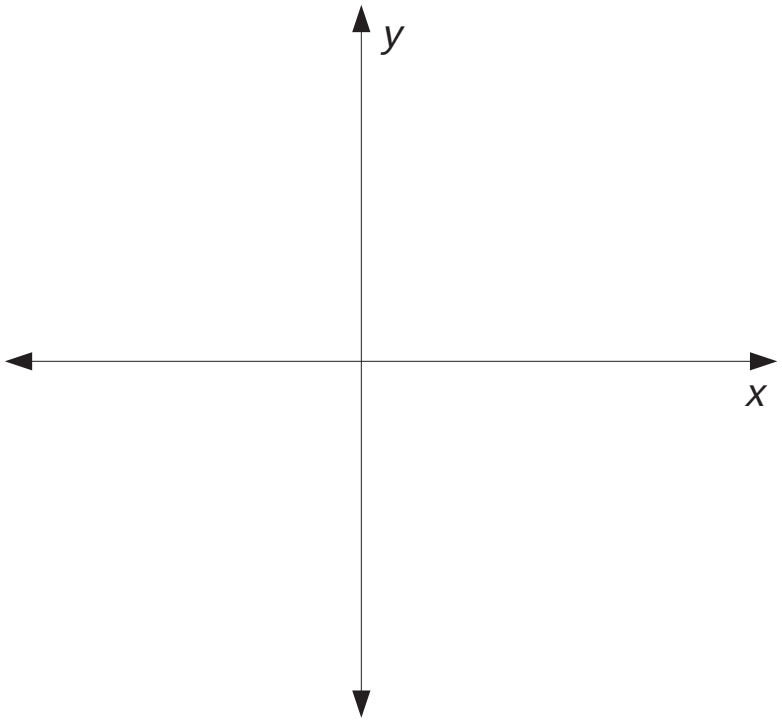
$$\int \left(e^{ix} + e^{-ix}\right)\, dx.$$

Write the result using only real-valued functions; the resulting expression should not contain the imaginary number i.

4. (8 points) Find the length of the curve $y = \int_1^x \sqrt{3 - t^2}\, dt$ for $0 \leq x \leq \sqrt{3}$.

5. (8 points) Consider the curve described by the equation $x^2 + 3y^2 = 1$.

(a) Carefully sketch the curve on the axes provided below. Be sure to label the x- and y-intercepts with their coordinates on your graph.



(b) Find the volume of the solid of revolution formed by rotating the curve about the x-axis.

6. (8 points) Consider the initial value problem $\begin{cases} y' = xy - x \\ y(1) = 0 \end{cases}$

(a) Use Euler's method with step size 0.2 to estimate $y(1.4)$.

(b) Solve the initial value problem explicitly. Use the solution to find the exact value of $y(1.4)$; you need not simplify the resulting expression.

7. (8 points) Evaluate the definite integral

$$\int_{-1}^{1} \frac{5x - 13}{(x + 3)(x^2 + 5)}\, dx.$$

8. (4 points) Evaluate the indefinite integral

$$\int x \sin(x) \cos^2(x)\, dx.$$

9. (6 points) Let $f(t) = te^{-3t}$ and for $x > 0$, let $g(x)$ be the average value of f on the interval $[0, x]$.

   (a) Express $g(x)$ as an integral.

   (b) Evaluate $g(2)$.

10. (6 points) Find the area enclosed by the polar curve $r(\theta) = \sin(\theta) + \sin(3\theta)$ that lies in the sector $0 \leq \theta \leq \frac{\pi}{2}$.