Print your name: ____________________ **Sec. Number:** ____
TA Name: ______________________ **Sec. Time:** ____

Math 20C
Quiz 3
November 9, 2005

1. (Sec. 14.6, Probl. 19) Find the directional derivative of $f(x,y) = \sqrt{xy}$ at $P(2,8)$ in the direction of $Q(5,4)$.

The unit vector $\mathbf{u}$ in the direction form P to Q is computed as follows.

$$\mathbf{v} = \vec{PQ} = \langle 5-2, 4-8 \rangle = \langle 3, -4 \rangle.$$

$$|\mathbf{v}| = \sqrt{9+16} = 5, \quad \Rightarrow \mathbf{u} = \frac{1}{5}\langle 3, -4 \rangle.$$

The gradient of f is given by

$$\nabla f(x,y) = \left\langle \frac{y}{2\sqrt{xy}}, \frac{x}{2\sqrt{xy}} \right\rangle = \frac{1}{2}\left\langle \sqrt{\frac{y}{x}}, \sqrt{\frac{x}{y}} \right\rangle.$$

Then,

$$\nabla f(2,8) = \frac{1}{2}\left\langle 2, \frac{1}{2} \right\rangle = \left\langle 1, \frac{1}{4} \right\rangle.$$

Therefore,

$$D_u f(2,8) = \left\langle 1, \frac{1}{4} \right\rangle \cdot \frac{1}{5}\langle 3, -4 \rangle = \frac{1}{5}(3-1),$$

then,

$$D_u f(2,8) = \frac{2}{5}.$$

2. (Sec. 14.7, Probl. 45) Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane

$$x + 2y + 3z = 6.$$

$$V(x, y, z) = xyz, \qquad \text{with } x > 0, \quad y > 0, \quad z > 0.$$

The constraint is $x + 2y + 3z = 6$. Then,

$$x = 6 - 2y - 3z, \quad \Rightarrow V(y, z) = 6yz - 2y^2z - 3yz^2.$$

Then,

$$V_y = 6z - 4yz - 3z^2 = 0, \quad z \neq 0, \quad \Rightarrow 6 - 4y - 3z = 0.$$
$$V_z = 6y - 2y^2 - 6yz = 0, \quad y \neq 0, \quad \Rightarrow 6 - 2y - 6z = 0.$$

The first equation says $3z = 6 - 4y$, then this expression into the second equation above implies

$$6 - 2y - 2(6 - 4y) = 0, \quad \Rightarrow -6 + 6y = 0, \quad \Rightarrow y = 1.$$

Then,

$$z = \frac{2}{3}, \qquad \text{and then } x = 6 - 2 - 3\,\frac{2}{3}, \quad \Rightarrow x = 2.$$

Finally,

$$V = \frac{4}{3}.$$