Name: ______________________________

(Use capitals)

Student number: ______________________________

Math 20C
Final Exam
July 31, 2004

Read each question carefully, and answer each question completely.
Show all of your work. No credit will be given for unsupported answers.
Write your solutions clearly and legibly. No credit will be given for illegible solutions.

1. (8 points)

   (a) Find the angle between the planes $2x + y + 3z = 1$ and $-x - 3y + 2z = 5$.

   (b) Find a vector parallel to the line of intersection of the planes given in (a).

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2. (8 points)

A particle has the velocity function $\vec{v}(t) = \langle -3\sin(t), 4, 3\cos(t) \rangle$.

(a) Find the particle acceleration $\vec{a}(t)$.

(b) The particle initial position is given by $\vec{r}(0) = \langle 3, 1, 0 \rangle$. Find the particle position function $\vec{r}(t)$.

(c) Reparametrize the curve $\vec{r}(t)$ with respect to the arc length measured from the point where $t = 0$, in the direction of increasing t.

3. (8 points)

(a) Find an equation for the plane tangent to the graph of $f(x, y) = \frac{1}{\pi} \cos\left(\frac{\pi}{2} x^2 y\right)$ at the point $(-1, 1)$.

(b) Find the linear approximation for $f(-1.1, 1.2)$.

4. (8 points)

Consider the function $f(x, y, z) = 2\sin(xyz)$.

(a) Find a unit vector in the direction of greatest increase of f at the point $(\pi, 1, 1)$.

(b) Find the directional derivative of f at the point $(\pi, 1, 1)$ in the direction given by the vector $\langle 1, 1, -1 \rangle$.

5. (8 points)

Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = xy$ subject to the constraint $2x + y^2 = 3$.

6. (8 points)

(a) Sketch the region of integration, D, whose area is given by the double integral $\int\int_D dA = \int_0^2 \int_{\frac{3}{2}x}^3 dy\, dx$.

(b) Compute the double integral given in (a).

(c) Change the order of integration in the integral given in (a). (You don't need to compute the integral again.)

7. (8 points)

Use polar coordinates to compute the double integral of $f(x, y) = xy$ in the region $D = \{(x, y) : \quad 0 \leq x, \quad x^2 + y^2 \leq 4\}$.

8. (8 points)

Use a triple integral to compute the volume of the tetrahedron whose sides are given by the planes $x = 0$, $y = 0$, $z = 0$, and $2x + 2y + z = 2$.