Global existence and exponential decay for hyperbolic dissipative relativistic fluid theories

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We consider dissipative relativistic fluid theories on a fixed flat, globally hyperbolic, Lorentzian manifold ($\mathbf{R} \times \mathbf{T}^3, g_{ab}$). We prove that for all initial data in a small enough neighborhood of the constant equilibrium states (in an appropriate Sobolev norm), the solutions evolve smoothly in time forever and decay exponentially to some, in general undetermined, constant equilibrium state. To prove this, three conditions are imposed on these theories. The first condition requires the system of equations to be symmetric hyperbolic, a fundamental requisite to have a well posed and physically consistent initial value formulation. For the flat space-times considered here the equilibrium states are constant, which is used in the proof. The second condition is a generic consequence of the entropy law, and is imposed on the non-principal part of the equations. The third condition is imposed on the principal part of the equations and it implies that the dissipation affects all the fields of the theory. With these requirements we prove that all the eigenvalues of the symbol associated to the system of equations of the fluid theory have strictly negative real parts, which, in fact, is an alternative characterization for the theory to be totally dissipative. Once this result has been obtained, a straightforward application of a general stability theorem due to Kreiss, Ortiz, and Reula implies the results mentioned above. © *1997 American Institute of Physics.*
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I. INTRODUCTION

In recent years there has been a substantial improvement of our understanding on how a proper description of dissipative fluids can be incorporated in the framework of the theory of relativity. Dissipative relativistic fluid theories satisfying an entropy law and having a well posed (symmetric hyperbolic) and causal initial value formulation have been presented.[1–3]

An important result on the physical meaning of all these hyperbolic theories was obtained recently.[4,5] It was shown that certain constitutive relations between the variables in the hyperbolic fluid theories, which have a clear physical meaning, approach in their time evolution the values predicted by the simplest covariant generalizations of the Navier–Stokes fields. This result is based on a fundamental hypothesis, namely that the solution of the hyperbolic fluid field equations exists and remains smooth and small during a long enough time interval, such that the relaxation to near Navier–Stokes behavior occurs. This is a very important check for these fluid theories, since at microscopic scales they are substantially different from the usual Navier–Stokes theories in the following sense: for the last ones, one expects to have smooth global solutions for all smooth initial data sets, as has been proved in lower dimensions,[6] while for the former ones, one

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expects the developing of discontinuities in the form of shock waves for crispy enough initial data. Thus, we can only hope to find a neighborhood of equilibrium data for which global solutions exist and so where the departures from Navier-Stokes are uniformly small.

The purpose of the present work is to look for conditions under which the fundamental hypothesis mentioned above is satisfied. To this end, we apply a theorem[7] which is a generalization to the case of partial differential equations of the Ljapunov stability theorem for ordinary differential equations. This generalization holds for hyperbolic systems such that the eigenvalues of their associated symbols have all strictly negative real parts which, as we shall see, is the case for the hyperbolic dissipative fluids.

In order to apply this general stability theorem, three conditions are imposed on the dissipative hyperbolic fluid theories. The first condition requires the symmetric hyperbolicity of the system of equations, a fundamental requisite to have a well posed and physically consistent initial value formulation. As we shall see, the symmetry is an automatic property of these theories, but the hyperbolicity has to be required. We also include in this condition that the space-time manifold is $\mathbf{R} \times \mathbf{T}^3$ with a flat metric; this is the most restrictive assumption, in particular it implies that equilibrium solutions are constant solutions. The other two conditions are of a generic type in the sense that all fluid systems, except for very specific and isolated ones, satisfy them. Specifically, the second condition requires that the nonprincipal part of the system of equations, which is responsible for the dissipation, satisfy certain negative-semidefiniteness condition. This condition assures that all perturbations to an equilibrium state, which are not tangent to the equilibrium submanifold, do dissipate towards equilibrium, and this is manifested by the fact that they make a positive definite contribution to the entropy. The third condition is a requirement on the principal part of the system of equations, and it means that the presence of dissipation affects all the fields of the theory, in the sense of not allowing for a decoupled set of fields with its own evolution not being driven by dissipation. Both the second and the third conditions have already been required in the literature with the aim of characterizing the equilibrium states.

These conditions allow us to apply the theorem proved in Ref. 7, which implies not only the global existence of solutions, but also their exponential decay to constant equilibrium for initial data near enough, in some appropriate norm, to constant equilibrium data.

The plan of the paper is as follows: In Sec. II we briefly introduce the fundamental aspects of the fluid theories, and state in detail the character of the conditions we impose on them. In Sec. III we state and prove our main result. Finally, in Sec. IV we present the conclusions.

II. DISSIPATIVE RELATIVISTIC FLUID THEORIES

In this section, following Ref. 3, we introduce the dissipative relativistic fluid theories. After introducing them, we describe the properties of these fluids needed to prove stability.

We assume that a fluid state is characterized by a finite collection of space-time tensor fields. Let φ^A denote these fields, where upper case indices stand for the entire set of tensor indices represented in this collection of fields. So we refer to φ^A as a point in the space of fluid states $\mathscr{S}$. Lower case indices will denote space-time indices. Repeated indices indicate contraction as usual in the abstract index notation. We restrict consideration to the fluid theories in which the field equations take the form

$$M_{AB}^a \nabla_a \varphi^B = -I_{AB} \varphi^B, \tag{1}$$

where M_{AB}^a and I_{AB} are smooth functions of the fluid fields φ^A and the space-time metric.

We say that system (1) is symmetric hyperbolic if $M_{AB}^a = M_{(AB)}^a$ and there exists a timelike future-directed u^a such that $N_{AB} \equiv -u_a M_{AB}^a$ is positive definite. This is a sufficient condition to have a well posed initial value formulation.

On physical grounds, to have as maximum speed of propagation the speed of light, the stronger condition of causality, that is, $-u_a M_{AB}^a$ is positive definite for all future-directed timelike

vector u^a, is usually required. This means that the characteristic surfaces of system (1) are inside the null cone given by the space-time metric. To prove global existence and decay only the weaker condition of hyperbolicity is needed.

We analyze now the structure of the equilibrium states. Following Refs. 2 and 3 we say that a fluid state φ^A, solution of the dynamical system of equations, is a *strict equilibrium state* if its time reverse is also a solution. We denote a strict equilibrium state by φ_0^A, and assign a subindex zero to any tensor evaluated at a strict equilibrium state. This definition implies that $I_{0AB}\varphi_0^B = 0$. More generally, we say the φ^A is a *momentary equilibrium state* if $I_{AB}\varphi^B = 0$. These states are called equilibrium states because their entropy production vanishes, and momentary because if this condition holds at a certain time, it will not necessarily hold in subsequent times (see Ref. 4). Every fluid state can be written as $\varphi^A = \psi^A + \eta^A$, where $I_{AB}\psi^B = 0$, and η^A is such that $I_{AB}\eta^B = 0$ implies $\eta^A = 0$. We call ψ^A the momentary equilibrium part of the fluid state.

Below we state the three conditions imposed on these fluid theories that will be used to show global existence and exponential decay.

(1) The fluid system of equations (1) is symmetric hyperbolic and the space-time is $(\mathbf{R} \times \mathbf{T}^3, g_{ab})$, where $\mathbf{T}^3$ denotes a three dimensional torus and g_{ab} is a flat metric.
(2) The tensor I_{0AB} must be symmetric and positive semidefinite.
(3) The map $\mathscr{F}_K : \mathscr{S}_\psi \to \mathscr{S}^*$ defined by $\mathscr{F}_K(\psi^A) \equiv K_a M_{0AB}^a \psi^B$ is injective for all space-time vectors $K^a \neq 0$, where $\mathscr{S}^*$ denotes the dual of the space of fluid states and $\mathscr{S}_\psi$ the subspace of momentary equilibrium states.

The first condition is the more restrictive and more work has to be done in order to weaken it. It would be interesting to treat physically relevant boundary conditions and arrive at similar results. The other two conditions are not very restrictive and all fluid systems, except for very specific and isolated ones, satisfy them.

The second requirement ensures that the effect of I_{AB} in equation (1) is to dissipate, in the sense of tending to move nonequilibrium states towards equilibrium as time grows. This is just a little stronger than the entropy condition in these fluid theories that requires the entropy source to be non-negative. This stronger condition was already considered in Ref. 2.

The third requirement is on the principal part of the equations. It implies that dissipation affects all the fields of the theory, in the sense of not allowing the existence of a decoupled subset of fields with its own evolution not being affected by dissipation. This requirement turns out to be equivalent (see Appendix A), at least for the case of divergence type fluid theories, to one assumed in the literature[2] to characterize equilibrium states. When condition 1 above holds, equilibrium states turn out to be constant states.[8]

III. GLOBAL EXISTENCE AND EXPONENTIAL DECAY

In the theorem below we present our main result about global existence in time and decay to strict constant equilibrium.

Theorem 1: Consider the Cauchy problem for system (1), corresponding to a hyperbolic divergence type fluid theory satisfying conditions 1–3. If the initial data is smooth and close enough in a $H^p(\mathbf{T}^3)$ Sobolev norm ($p > 5$) to the data corresponding to some constant (strict equilibrium) solution, then the solution is smooth, exists globally in time and decays exponentially to some constant (strict equilibrium) solution in the H^p norm.

Proof: We define $N_{AB} = -u_a M_{AB}^a$, $N_{AB}^a = -q_b^a M_{AB}^b$ where $q_{ab} = g_{ab} + u_a u_b$ is the 3-metric in each hypersurface orthogonal to u^a (assuming $u^a u_a = -1$). Assume that the initial data is close to some strict equilibrium constant state φ_0^A. Then the fluid state is $\varphi^A = \varphi_0^A + \delta\varphi^A$. For the variable $\delta\varphi^A$, the system (1) becomes

$$N_{AB} u^a \nabla_a \delta\varphi^B = N_{AB}^a \nabla_a \delta\varphi^B - I_{AB} \delta\varphi^B. \tag{2}$$

Here the tensors N_{AB}, N_{AB}^a, and I_{AB} are thought of as functions of $\delta\varphi^A$. The condition (1) of Sec. II on the fluid theories implies that we can choose Cartesian coordinates $\{t, x^j\}$ on the space-time manifold $\mathbf{R} \times \mathbf{T}^3$ and a constant u^a such that $\partial/\partial t = u^a \nabla_a$. Then the system (2) becomes

$$N_{AB} \frac{\partial}{\partial t} \delta\varphi^B = N_{AB}^j \frac{\partial \delta\varphi^B}{\partial x^j} - I_{AB} \delta\varphi^B. \tag{3}$$

In order to study solutions near $\delta\varphi^A = 0$, it is convenient to introduce a parameter ε to control the smallness of initial data. Thus, $\delta\varphi^A(t=0) = \varepsilon f^A(x^j)$ and the solution shall be written as $\delta\varphi^A = \varepsilon v^A$. As the tensors N_{AB}^j and I_{AB} are smooth functions of εv^A, they can be written as

$$N_{AB}^j = N_{0AB}^j + \varepsilon N_{1AB}^j, \quad I_{AB} = I_{0AB} + \varepsilon I_{1AB}.$$

Notice that the tensors N_{0AB}^j and I_{0AB} are constant tensors since they are evaluated at constant states. With this decomposition the Cauchy problem for (3) is

$$N_{AB} \frac{\partial v^B}{\partial t} = (N_{0AB}^j + \varepsilon N_{1AB}^j) \frac{\partial v^B}{\partial x^j} + (I_{0AB} + \varepsilon I_{1AB}) v^B, \quad v^A(t=0) = f^A(x^j). \tag{4}$$

As there are periodic boundary conditions on the space coordinates $\{x^j\}$, v^A can be expanded in Fourier series,

$$v^A = \sum_{k^j \in \Omega} \hat{v}^A(k^j, t) e^{i\vec{x} \cdot \vec{k}},$$

where Ω is the discrete set of Fourier frequencies.

We want to apply the stability theorem proved in Ref. 7 that, for completeness, we state in Appendix B. To prove Theorem 1 we consider the eigenvalues problem

$$\lambda N_{0AB} \hat{\varphi}^B = (ik_j N_{0AB}^j - I_{0AB}) \hat{\varphi}^B. \tag{5}$$

Then, as explained in Appendix B, we only need to verify the following conditions.

(i) There is a constant $\delta > 0$ such that the eigenvalues $\lambda(k^j)$ satisfy $\text{Re}\{\lambda\} \leq -\delta$ for all $k^j \in \Omega$, $k^j \neq 0$.

(ii) For $k^j = 0$, $\lambda(0) \leq -\delta$ or $\lambda(0) = 0$.

(iii) As linear maps, the null space of I_{1AB} contains the null space of I_{0AB}.

Conditions (ii) and (iii) are satisfied for these fluid theories, since the kernel of I_{AB} is of constant dimension (see Refs. 2 and 3), and because condition (2) of Sec. II holds.

To prove (i), let k^j be different from zero. Then $k = \sqrt{k^j k_j} \geq \text{const.} > 0$. The eigenvalue problem (5) can be written as

$$\left(-\frac{\lambda}{k} N_{0AB} + i \frac{k_j}{k} N_{0AB}^j \right) \hat{\psi}^B = \left(\frac{\lambda}{k} N_{0AB} + \frac{1}{k} I_{0AB} - i \frac{k_j}{k} N_{0AB}^j \right) \hat{\eta}^B.$$

Defining $K_a = -(\lambda/k) u_a + i(k_j/k) q_a^j$, this can be written as

$$K_a M_{0AB}^a \hat{\psi}^B = \left(\frac{\lambda}{k} N_{0AB} + \frac{1}{k} I_{0AB} - i \frac{k_j}{k} N_{0AB}^j \right) \hat{\eta}^B. \tag{6}$$

Since $K_a M_{0AB}^a$ is injective, by condition (3), and a smooth function of k_j/k, there is a constant $c > 0$ such that

$$cN_{0BC}\overline{\hat{\psi}}^B\hat{\psi}^C \leq K_a M_0^{aA}{}_B K_b M_{0AC}^b \overline{\hat{\psi}}^B \hat{\psi}^C.$$

Then, contracting (6) with itself,

$$N_{0BC}\overline{\hat{\psi}}^B\hat{\psi}^C \leq \frac{1}{c}\left(\frac{\overline{\lambda}}{k}N_0^A{}_B + \frac{1}{k}I_0^A{}_B + i\frac{k_j}{k}N_0^{jA}{}_B\right)\left(\frac{\lambda}{k}N_{0AC} + \frac{1}{k}I_{0AC} - i\frac{k_j}{k}N_{0AC}^j\right)\overline{\hat{\eta}}^B\hat{\eta}^C.$$

Perturbation theory of linear operators tells us that $\lambda(k^j)/k$ is uniformly bounded (see Ref. 9). Then we get for some positive c'

$$N_{0BC}\overline{\hat{\psi}}^B\hat{\psi}^C \leq c'N_{0BC}\overline{\hat{\eta}}^B\hat{\eta}^C.$$

This inequality, together with the positive definiteness of N_{0AB}, implies

$$N_{0AB}\overline{\hat{\varphi}}^A\hat{\varphi}^B \leq \widetilde{c}N_{0AB}\overline{\hat{\eta}}^A\hat{\eta}^B, \quad \widetilde{c} > 0. \tag{7}$$

Now, contracting (5) with $\overline{\hat{\varphi}}^A$ and taking the real part,

$$\mathrm{Re}\{\lambda\}N_{0AB}\overline{\hat{\varphi}}^A\hat{\varphi}^B = -I_{0AB}\overline{\hat{\varphi}}^A\hat{\varphi}^B = -I_{0AB}\overline{\hat{\eta}}^A\hat{\eta}^B$$

$$\leq -\widetilde{\delta}N_{0AB}\overline{\hat{\eta}}^A\hat{\eta}^B$$

$$\leq -\frac{\widetilde{\delta}}{\widetilde{c}}N_{0AB}\overline{\hat{\varphi}}^A\hat{\varphi}^B.$$

Here, $\widetilde{\delta} > 0$ exists because of the negative definiteness of I_{0AB} in the direction of $\hat{\eta}^A$ (by condition (2)), and we have used (7) in the last line. We have thus shown that

$$\mathrm{Re}\{\lambda(k^j)\} \leq -\delta < 0, \quad \text{with } \delta = \frac{\widetilde{\delta}}{\widetilde{c}} > 0 \text{ and } k^j \neq 0,$$

and Theorem 1 is proved.

IV. CONCLUSIONS

In this work we have proved global existence and exponential decay to constant strict equilibrium states, for solutions of a generic dissipative relativistic fluid theory. These decaying solutions correspond to initial data in a small enough neighborhood of constant strict equilibrium.

This result, in particular, verifies a fundamental hypothesis of previous works,[4,5] namely, the existence of solutions during a long enough time interval. The closeness of initial data to strict equilibrium is a natural limitation in the sense that, for large data, shock waves develop, which are a widely observed phenomena in nature. This occurs because these fluid systems are genuinely non-linear.

There are three questions related to the techniques used in this work and the possibility of improving on them. One is whether it is possible to extend the present result, or rather the general theorem used, to the case of non-constant equilibrium states. This is of vital importance if we want to consider non-flat backgrounds or even self-gravitating fluids. We think this is probably the case if we further assume that the theory has, at equilibrium, a conserved energy—which is a positive definite bilinear form in the tangent space of equilibrium states—as is usually the case for theories coming from a Hamiltonian formalism. The second possible extension is towards allowing non-

compact Cauchy slices. There is another technique, introduced by Matsumura,[10] which allows us to study global existence and stability for some particular cases that range from hyperbolic heat conduction to relativistic superfluids.[11] This technique allows us to treat the case of non-compact Cauchy slices, but can not be applied to the general systems considered in this work. Thus, it would be important to extend the theorem given in Appendix B to the case of non-compact Cauchy slices. The third extension is in the direction of boundary values problems. It is clear that one would like to use this theory to describe situations where the fluid is in a finite region of space, in which case the equations cease to be hyperbolic outside the region occupied by the fluid, and so a boundary value formulation is needed. Since Navier–Stokes fluids behave in a much more amenable way with regards to boundary conditions than perfect fluids, one would expect that these dissipative fluids will have that property too, making this an interesting, and perhaps tractable, problem.

ACKNOWLEDGMENT

APPENDIX A: EQUIVALENCE OF ASSUMPTIONS

In this Appendix we prove the equivalence between the requirement (3) of Sec. II and a condition imposed in Refs. 2 and 3. We do this, for simplicity, in the case of divergence type fluid theories, and we assume that the reader is familiar with Ref. 2. The condition under consideration allows us to characterize the strict equilibrium states in such a way that they are precisely the same set of equilibrium states found for the standard Eckart theory.

The requirement 3 of Sec. II is the following: The map $\mathscr{F}_K : \mathscr{S}_\psi \to \mathscr{S}^*$, defined by $\mathscr{F}_K(\psi^A) \equiv K_a M^a_{0AB} \psi^B$, is injective for all space-time vectors $K^a \neq 0$, where $\mathscr{S}^*$ denotes the dual of the space of fluid states and $\mathscr{S}_\psi$ the subspace of momentary equilibrium fluid states.

The equivalence between this condition and the one assumed in Ref. 2 follows from the following argument. The map $\mathscr{F}_K$ is injective, so $K_a M^a_{0AB} \psi^B = 0 \Rightarrow \psi^A = 0$ where $\psi^A = (\psi, \psi^a, 0)$. Due to the definition of indices A and B, the system of equations above represents a scalar equation, a vector equation, and a symmetric two-index tensor equation.

First, consider the scalar equation, the contraction of the vector equation with ζ^a, and the contraction of the two-index tensor equation with $\zeta^a \zeta^b$. All this constitutes a linear algebraic system of three scalar equations for variables $K_a \psi^a$, $K_a \zeta^a \psi$, and $2 K_a \zeta^a \zeta^b \psi_b$. The injectivity implies that the only solution for these three variables is zero and so the determinant of the coefficient matrix is different from zero. Conversely, if the determinant of the coefficient matrix is different from zero, then we conclude that $\psi = 0$ and $\psi^a = 0$ and then the map $\mathscr{F}_K$ is injective. By direct inspection it can be checked that these coefficients are the same found in equations (41)–(43) in Ref. 2.

Second, consider the vector equation and the contraction of the two-index tensor equation with ζ^a. This constitutes a linear algebraic system of two vector equations for the variables mentioned in paragraph above, and $2 \zeta^a K_{(a} \psi_{b)}$ and $K_b \psi$. Because of injectivity, the only solution for all these variables is zero and so the determinant of the coefficient matrix is different from zero. It can be checked, by direct inspection, that these coefficients are the same found in equations (45)–(46) in Ref. 2.

Finally, consider the two-index tensor equation. It constitutes a linear algebraic two-index equation for the variables mentioned in the previous paragraph and $K_{(a} \psi_{b)}$. Because of the injectivity, the only solution for all these variables is zero and so the determinant of the coefficient matrix is different from zero. By direct inspection it can be checked that these coefficients are the same found in equations (48) in Ref. 2.

APPENDIX B: GENERAL STABILITY THEOREM

Consider the Cauchy problem for a first order system of partial differential equations,

$$\frac{\partial v}{\partial t} = (A_0^j + \varepsilon A_1^j(v, \varepsilon)) \frac{\partial v}{\partial x^j} + (B_0 + \varepsilon B_1(v, \varepsilon)) v,$$

(B1)

$$v(t = 0) = f(x^j),$$

where $v: \mathbf{R} \times \mathbf{T}^s \to \mathbf{R}^n$, $A_1^j(v, \varepsilon)$ and $B_1(v, \varepsilon)$ are smooth (C^∞) functions of their arguments, and $f(x): \mathbf{T}^s \to \mathbf{R}^n$ is also smooth. Let P denote the projector in the kernel of B_0. For the solution v of (B1) we define $v^{(0)} = P\hat{v}(0, t)$ and $w = v - v^{(0)}$. The stability theorem proved in Ref. 7 states:[12]

Theorem 2: *Suppose that the matrices A_0^j, B_0, and A_1^j are Hermitian, and the system (B1) satisfy the ''relaxed stability eigenvalue condition,'' i.e., the following conditions hold.*

- (i) *There is a constant $\delta > 0$ such that the eigenvalues $\lambda(k)$ of the symbol $iA_0^j k_j + B_0$ satisfy $\mathrm{Re}\{\lambda\} \leqslant -\delta$ for all $k \in \Omega$, $k \neq 0$.*
- (ii) *The eigenvalues of B_0 satisfy either $\mathrm{Re}\{\lambda(0)\} \leqslant -\delta$ or $\lambda(0) = 0$.*
- (iii) *$\ker B_0 \subset \ker B_1$.*

Then, for $0 \leqslant \varepsilon \leqslant \varepsilon_0$ with ε_0 small enough, the system (B1) is a contraction for w in a suitable norm, equivalent to a Sobolev norm H^p ($p > s + 2$), and $v^{(0)} \to$ const. when $t \to \infty$.

The statement in the above theorem that the system is contraction means that there exists an H-norm, equivalent to the norm H^p, such that

$$\frac{d}{dt} \|w\|_H^2 \leqslant -(\delta + \mathcal{O}(\varepsilon)) \|w\|_H^2.$$

This implies that, for ε small enough, there exists a global (in time) smooth solution of the Cauchy problem (B1) such that there is an exponentially decaying bound for the Sobolev H^p norm of the w-part of its solution, and the $v^{(0)}$ part goes to a constant when the time goes to infinity.

The only difference between the fluid equations (4) and (B1) is the presence of N in front of the time derivative. This causes no difficulties, and the theorem above is applicable by a simple redefinition of the scalar product used. With this new scalar product, the eigenvalues problem for the fluid equations becomes

$$\det(iN_0^{-1} N_0^j k_j + N_0^{-1} I_0 - \lambda E) = 0,$$

where E is the identity matrix. Then, the conditions (i), (ii), and (iii) of Theorem 2 can be verified as is done in the proof of Theorem 1.

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