MTH 320: Homework 7

The following are due on Monday, February 25:
§3.2:

1. Let
$$B = \left\{ \frac{n(-1)^n}{n+1} : n \in \mathbb{N} \right\}.$$
   (a) Find the limit points of B.
   (b) Is B a closed set?
   (c) Is B an open set?
   (d) What are the isolated points of B?
   (e) Find $\bar{B}$.

2. Let (a_n) be a sequence of points in $A \setminus \{x\}$ such that $\lim a_n = x$. Prove that x is a limit point of A.

3. (a) If y is a limit point of $A \cup B$, show that y is a limit point of A or a limit point of B.
   (b) Prove $\overline{A \cup B} = \bar{A} \cup \bar{B}$.
   (c) Does (b) extend to infinite unions of sets? If not, find a counterexample.

4. True or false. If true, prove it, if false, provide a counterexample.
   (a) For any $A \subset \mathbb{R}$, the set $\mathbb{R} \setminus \bar{A}$ is open.
   (b) If a set A has an isolated point, it cannot be open.
   (c) A set A is open if and only if $\bar{A} \neq A$.
   (d) If A is bounded, $s = \sup A$ is a limit point of A.
   (e) Every finite set is closed.
   (f) An open set that contains every rational number must necessarily be all of $\mathbb{R}$.

§3.3: Compact sets are a generalization of finite sets in many ways. The following exercise shows how this generalization holds or fails.

5. True or false. If true, prove it, if false, provide a counterexample.
   (a) A finite union of compact sets is compact.
   (b) A finite set is always compact.
   (c) A countable set is always compact.
   (d) If K is compact, then $\sup K$ is contained in K.
   (e) Any subset of a compact set is compact.