A solution from Homework 1

§1.3.38:

Analytically, we can solve for the angle θ between $\mathbf{a}$ and $\mathbf{x}$ in terms of $\|\mathbf{a}\|$ and $\|\mathbf{b}\|$. We know that

$$\|\mathbf{a}\| = \mathbf{x} \cdot \mathbf{a} = \|\mathbf{x}\|\|\mathbf{a}\| \cos(\theta),$$

and

$$\|\mathbf{b}\| = \|\mathbf{x} \times \mathbf{a}\| = \|\mathbf{x}\|\|\mathbf{a}\| \sin(\theta).$$

So

$$\|\mathbf{b}\|/\|\mathbf{a}\| = \tan(\theta).$$

The only case when the above division doesn't make sense is when $\|\mathbf{a}\| = 0$, so $\mathbf{a}$ is the zero vector. Indeed in this case $\mathbf{b}$ must also be the zero vector from the cross product information we were given, and in this case any $\mathbf{x}$ satisfies the given equations. Otherwise, $\theta = \arctan(\|\mathbf{b}\|/\|\mathbf{a}\|)$ and so we know the angle between $\mathbf{a}$ and $\mathbf{x}$.

Geometrically, the right hand rule tells us which way $\mathbf{x}$ is pointing with respect to $\mathbf{a}$ and $\mathbf{b}$; for example, if $\mathbf{a}$ is on the $y-$axis and $\mathbf{b}$ is on the $z-$axis, we know $\mathbf{x}$ must be in the $xy-$plane, since $\mathbf{b}$ is orthogonal to both $\mathbf{x}$ and $\mathbf{a}$ (and in fact must be in the first quadrant of the $xy-$plane since $\cos(\theta) \geq 0$).

Since we know the angle between $\mathbf{x}$ and $\mathbf{a}$, the length of $\mathbf{x}$ (this is not enough yet since $\mathbf{x}$ can be on the "right"or the "left"of $\mathbf{a}$), and the orientation of $\mathbf{x}$ with respect to $\mathbf{a}$, we know $\mathbf{x}$ completely.