Homework 7

The following are due on Monday, February 26:

§4.3 # 1, 9, 10, 13, 21.

§4.4 # 9, 10, 13, 14, 23

§5.1 # 1, 5, 7, 8, 13.

Extra Credit (recommended):

§4.2 # 20, 21(c), 23(a). Use the product rule for cross product: $(c(t) \times d(t))' = c'(t) \times d(t) + c(t) \times d'(t)$. For 23(a) use the formula for $d\mathbf{B}/dt$ given to you in 21.

More extra credit: Given a nowhere zero vector field $F : \mathbb{R}^3 \to \mathbb{R}^3$, consider

$$F \cdot (\nabla \times F).$$

Can you find an example for which this expression is not identically zero?

Prove the following: $F \cdot (\nabla \times F) = 0$ if there is some continuously differentiable nowhere zero function $g : \mathbb{R}^3 \to \mathbb{R}$ such that $\nabla \times (gF) = \mathbf{0}$. (By a Theorem from class, in other words, if gF is the gradient of some function $f : \mathbb{R}^3 \to \mathbb{R}$.)[1]

[1] The converse is also true, but much harder to prove. This question is motivated by the following: many students remember $\nabla \cdot \nabla \times F = 0$ by using the geometric interaction of dot and cross product of vectors $u \cdot u \times v = 0 = v \cdot u \times v$, since $u \times v$ is orthogonal to u and orthogonal to v. For any C^2 vector field F on $\mathbb{R}^3$ it's true that $\nabla \cdot \nabla \times F = 0$, but what about $F \cdot \nabla \times F$? As we can see from this exercise, there is a nice geometric property of F that makes this happen: this is true whenever F can be rescaled to be the gradient of some function.