Homework 4

The following are due on Monday, February 5:

§2.5 #2 (e) and (f), 3 (b) and (c), 5, 11,

Chain Rule: Assume $F(x, y, z) = 0$ is a level set of $F : \mathbb{R}^3 \to \mathbb{R}$. Note that this implicitly defines y as a function of x and z (if we move x, z a little bit, we have to compensate with y to keep F equal to 0, unless F is already identically zero). Using the chain rule, prove that

$$\frac{\partial y}{\partial x} = -\frac{\partial F/\partial x}{\partial F/\partial y}.$$

Similarly,

$$\frac{\partial x}{\partial z} = -\frac{\partial F/\partial z}{\partial F/\partial x} \text{ and } \frac{\partial z}{\partial y} = -\frac{\partial F/\partial y}{\partial F/\partial z}.$$

You can look at the hint to Leibniz's Rule problem below for a little extra help. Use the above to do exercise #20. Note that you can't cancel partials in higher dimensions, and this is the closest we can get to doing that.

#30.

Leibniz's Rule: Prove Leibniz's rule using the chain rule: if $a(x), b(x)$, and $f(x, y)$ are differentiable, then

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(x,t)dt = f(x, b(x))b'(x) - f(x, a(x))a'(x) + \int_{a(x)}^{b(x)} \frac{\partial f}{\partial x}(x,t)dt.$$

Hint: let $g(y, z) = \int_0^y f(z, t)dt$. Letting $y(x) = b(x)$ and $z(x) = x$, the chain rule tells us

$$\frac{\partial g}{\partial x}(b(x), x) = \frac{\partial g}{\partial y}\frac{dy}{dx}(b(x), x) + \frac{\partial g}{\partial z}\frac{dz}{dx}(b(x), x) = \frac{\partial g}{\partial y}\frac{dy}{dx}(b(x), x) + \frac{\partial g}{\partial z}(b(x), x).$$

Note that $\int_{a(x)}^{b(x)} f(x, t)dt = g(b(x), x) - g(a(x), x)$. Use the fundamental theorem of calculus for $\partial g/\partial b$ and $\partial g/\partial a$.

Assume you are allowed to do $\partial/\partial z \int_0^y f(z, t)dt = \int_0^y (\partial f/\partial z)(z, t)dt$.

§2.6 #22, 25, 32.

§3.1 #13, 15, 17 (this is a proof, so use Theorem 1 carefully), 25, 26, 32.

§3.2 #2, 5, 7.