Homework 12

The following are due on Monday, April 9:

§7.3 # 8, 20, extra credit: 24. Also (not extra credit):

A Given a continuously differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$, prove that $(u, v, f(u, v))$ is a regular surface.

B Find a parametrization of $x^2 - y^2 - z^2 = 1$ (without square roots). Is it regular?

C Definition: A *ruled surface* is one that can be parametrized by $r(u, v) = \boldsymbol{\alpha}(u) + v\boldsymbol{\beta}(u)$, where $\boldsymbol{\alpha} : [a, b] \to \mathbb{R}^3$ and $\boldsymbol{\beta} : \mathbb{R} \to \mathbb{R}^3$.

Question: Describe what a ruled surface is geometrically. Next, find $y(u, v)$ such that the parametrization

$$\mathbf{r}(u, v) = (\cos(u) - v\sin(u), y(u, v), v)$$

parametrizes $x^2 + y^2 - z^2 = 1$ and proves it is a ruled surface.

§7.4 # 5, 9, 23, extra credit: 18.