

**MTH 370, Fall 2009**  
**Homework 12**

**Instructions:** Do these calculations by hand (you may use a computer or calculator for simple arithmetic and function evaluations) and show your work.

1. Show that the two-species competition model

$$\begin{aligned}\frac{dx}{dt} &= r_1 x \left( 1 - \frac{x + \beta_{12} y}{\kappa_1} \right), \\ \frac{dy}{dt} &= r_2 y \left( 1 - \frac{y + \beta_{21} x}{\kappa_2} \right),\end{aligned}$$

has no limit-cycle solutions in the positive quadrant (i.e., when  $x > 0$ ,  $y > 0$ ). [Hint: Set  $h(x, y) = \frac{1}{xy}$  and use Dulac's negative criterion.]

2. Consider the following nondimensional model from the last homework:

$$\begin{aligned}\frac{du}{d\tau} &= c - u + u^2 v, \\ \frac{dv}{d\tau} &= d - u^2 v.\end{aligned}$$

Assuming that  $0 < c \ll d$ , argue that this system undergoes a Hopf bifurcation when  $d \approx 1$ .