MTH 370, Fall 2009
Homework 11

Instructions: Do these calculations by hand (you may use a computer or calculator for simple arithmetic and function evaluations) and show your work.

1. Consider the following reactions:

$$X \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} A, \quad B \overset{k_2}{\to} Y, \quad 2X + Y \overset{k_3}{\to} 3X$$

   (a) Write down the mass action equations for these reactions, treating the concentrations of A and B as positive constants.

   (b) Show that, by making the change of variables

$$u = \sqrt{\frac{k_3}{k_1}}x, \quad v = \sqrt{\frac{k_3}{k_1}}y, \quad \tau = k_1 t,$$

   the mass action equations of part (a) become

$$\begin{aligned} \frac{du}{d\tau} &= c - u + u^2 v \\ \frac{dv}{d\tau} &= d - u^2 v \end{aligned} \tag{1}$$

   where c and d are positive constants.

   (c) Show that the system (1) has exactly one equilibrium, that this equilibrium is positive, and that it is repelling if and only if

$$2d > (c+d)(1+(c+d)^2). \tag{2}$$

   (d) Assuming that the inequality (2) holds, show that the region D bounded by the four lines

$$u = c, \quad v = 0, \quad v = \frac{d}{c^2}, \quad v = \frac{d}{c^2} + c + d - u,$$

   is a trapping region for the solutions of (1).

   (e) Conclude that the region D contains a limit cycle.