MTH 370, Fall 2009
Homework 10

Instructions: Do these calculations by hand (you may use a computer or calculator for simple arithmetic and function evaluations) and show your work.

1. In the real world, trimolecular reactions are rare, although trimers are not. Consider the following trimerization reaction in which three monomers of X combine to form the trimer Z:

$$X + X \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} Y$$

$$X + Y \underset{k_{-2}}{\overset{k_2}{\rightleftarrows}} Z$$

   (a) Write down the mass-action equations for these reactions.

   (b) Show that
$$\frac{dx}{dt} + 2\frac{dy}{dt} + 3\frac{dz}{dt} = 0.$$
   Why does this equation hold?

   (c) Letting x_0 denote the initial concentration of monomers, suppose
$$k_{-1} \gg k_{-2},\ k_{-1} \gg k_2 x_0.$$
   Use a quasi-steady-state approximation to find the rate of production of Z, and show that it is proportional to x^3.

2. The length of a microtubule changes by a process called *treadmilling* – monomer is added to one end of the microtubule and removed from the other. A simple set of reactions for this process is the following. Let X_n denote a microtubule comprised of n monomers. We say that n is the *length* of such a microtubule. Then for each $n \geq 1$,
$$X_1 + X_n \underset{k_-}{\overset{k_+}{\rightleftarrows}} X_{n+1},$$
   where k_+ is the rate of adding monomer, and k_- is the rate of removing it. Assuming that there is a total concentration c of monomers, find the equilibrium distribution of microtubule lengths.