MTH 370, Fall 2009
Homework 9

Instructions: Do these calculations by hand (you may use a computer or calculator for simple arithmetic and function evaluations) and show your work.

1. Write down the mass-action equations for the following chemical reactions:

   (a) $X + Y \xrightarrow{k} 2Y$

   (b) $2X \xrightarrow{k} X + Y$

   (c) $2X + Y \xrightarrow{k} X + 2Y$

2. Consider the following chemical reactions:

$$A + X \xrightarrow{k_1} 2X$$
$$X + Y \xrightarrow{k_2} 2Y$$
$$Y \xrightarrow{k_3} \emptyset$$

   Assuming the concentration of A is kept constant, show that the mass-action equations for these reactions are the same as the Loktka-Volterra model for predator-prey systems. Which chemical is the "prey" and which is the "predator"?

3. Consider the following chemical reactions:

$$X \underset{\beta}{\overset{2\alpha}{\rightleftarrows}} Y \underset{2\beta}{\overset{\alpha}{\rightleftarrows}} Z$$

   (a) Write down the mass-action equations for the concentrations x, y and z, and show that $x + y + z$ is constant.

   (b) Show that when $x+y+z = 1$, the steady-state solution of the mass-action equations is a binomial distribution with parameter $p = \alpha/(\alpha + \beta)$.

   (c) What kind of biological system might these reactions describe?