

MTH 370, Fall 2009
Homework 8

Instructions: Do these calculations by hand (you may use a computer or calculator for simple arithmetic and function evaluations) and show your work.

1. One unrealistic feature of the the Lotka-Volterra model is that it the prey population grows without bound in the absence of predators. We can remedy this fault by introducing a logistic-type term for the prey's reproduction rate:

$$\begin{aligned}\frac{dx}{dt} &= (a(K - x) - by)x, \\ \frac{dy}{dt} &= (dx - c)y,\end{aligned}$$

where $K > 0$ is the carrying capacity for the prey population. Analyze this modified Lotka-Volterra model the same way we did the original Lotka-Volterra model in class. That is, find the equilibria and determine their stability type, find the x - and y -nullclines and determine in which direction solutions traverse them, and then plot all this information, including a few representative solutions, in the phase-plane.