

MTH 370, Fall 2009
Homework 7

Instructions: Do these calculations by hand (you may use a computer or calculator for simple arithmetic and function evaluations) and show your work.

1. Solve the following systems of first-order linear ODEs. In each problem, classify the type and stability of the origin.

(a) $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$, $A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$

(b) $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$, $A = \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix}$

(c) $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

2. Consider the system of first-order linear ODEs

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x}, \quad A = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \quad (1)$$

- (a) Solve (1) first by solving the second equation, and then plugging this into the first equation and solving it by integrating factors.
- (b) Now try to solve (1) by calculating e^{At} . If you have trouble, explain why. Can you calculate e^{At} directly from its Taylor series?