MTH 370, Fall 2009
Homework 4

Instructions: Do these calculations by hand (you may use a computer or calculator for simple arithmetic and function evaluations) and show your work.

1. Recall from class that the characteristic equation of the matrix A can be written

$$0 = \det(A - \lambda I) = \lambda^2 - \operatorname{tr}(A)\lambda + \det(A).$$

   (a) Letting λ_+ and λ_- denote the two eigenvalues of A, prove that

$$\operatorname{tr}(A) = \lambda_+ + \lambda_- \quad \text{and} \quad \det(A) = \lambda_+\lambda_-.$$

   [Hint: Use the fact that λ_+ and λ_- are the two roots of the quadratic polynomial $\det(A - \lambda I)$.]

   (b) Verify that (a) is indeed true for the matrix

$$A = \begin{bmatrix} 7 & 2 \\ 2 & 7 \end{bmatrix}.$$

2. Let

$$A = \begin{bmatrix} 0 & 3 \\ 3 & -1 \end{bmatrix}$$

   (a) Compute the inverse A^{-1} of A.

   (b) Solve the following matrix equation for $\mathbf{x}$:

$$A\mathbf{x} = \mathbf{b} = \begin{bmatrix} -9 \\ 18 \end{bmatrix}$$

3. Consider the age-structured population model from class,

$$\mathbf{x}_{n+1} = M\mathbf{x}_n \tag{1}$$

   where

$$M = \begin{bmatrix} p & q \\ k & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{x}_n = \begin{bmatrix} A_n \\ J_n \end{bmatrix}.$$

   (a) Show that when $p = 0.5$, $q = 0.1$ and $k = 2$, the population will go extinct, no matter the initial size of the population.

   (b) Setting $p = 0.5$ and $k = 2$, find the critical value q_c such that if $q > q_c$ then the population grows, but if $q < q_c$ then the population goes extinct.