MTH 370, Fall 2009
Solutions to Homework 3

1. Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

   (a) Show that AB does not equal BA.

   (b) Find a matrix $C \neq A$ that commutes with A, that is, $AC = CA$.

   Solution:

   (a)

$$AB = \begin{bmatrix} 4 & 6 \\ 4 & 6 \end{bmatrix} \neq \begin{bmatrix} 3 & 3 \\ 7 & 7 \end{bmatrix} = BA$$

   (b) Any multiple $C = mI$ of the identity matrix works:

$$AC = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} = \begin{bmatrix} m & m \\ m & m \end{bmatrix} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = CA$$

2. Let

$$A = \begin{bmatrix} 7 & 2 \\ 2 & 7 \end{bmatrix}$$

   Find the eigenvectors and associated eigenvalues of A.

   Solution: The characteristic equation is

$$0 = \det(A - \lambda I) = \lambda^2 - 14\lambda + 45 = (\lambda - 9)(\lambda - 5)$$

   hence the eigenvalues are $\lambda_+ = 9$ and $\lambda_- = 5$. The associated eigenvectors are

$$\mathbf{x}_+ = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{x}_- = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

3. Let

$$A = \begin{bmatrix} 0.5 & -2 \\ 0 & 0.9 \end{bmatrix}$$

   Find the eigenvectors and associated eigenvalues of A.

   Solution: The characteristic equation is

$$0 = \det(A - \lambda I) = \lambda^2 - 1.4\lambda + 0.45 = (\lambda - 0.9)(\lambda - 0.5)$$

   hence the eigenvalues are $\lambda_+ = 0.9$ and $\lambda_- = 0.5$ (notice these are the same as the diagonal entries). The associated eigenvectors are

$$\mathbf{x}_+ = \begin{bmatrix} -5 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{x}_- = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$