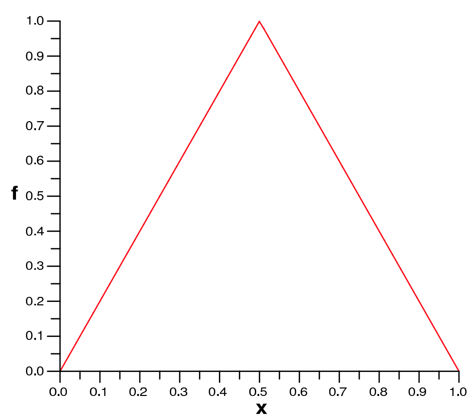
MTH 370, Fall 2009
Homework 2

Instructions: Do these calculations by hand (you may use a computer or calculator for simple arithmetic and function evaluations) and show your work.

1. Does the difference equation
$$c_{n+1} = c_n(3.5 - 35c_n)$$
have a solution of period 2? If so, is it stable? [Hint: there is a hard and easy way to do this problem. The hard way is to do all the calculations that we did in class for this particular difference equation. The easy way is to make a change of variables that puts this difference equation into the form of the discrete logistic equation that we studied in class, $b_{n+1} = rb_n(1 - b_n)$, and then answering the question given the value of r.]

2. Consider again the nonlinear difference equation
$$c_{n+1} = rc_n \mathrm{e}^{-c_n}.$$
Find the point r_2 such that solutions of period 2 exist for $r \geq r_2$ and do not exist for $r < r_2$.

3. Consider the following one-dimensional nonlinear difference equation,
$$x_{n+1} = f(x_n), \tag{1}$$
where f is the *tent map*,
$$f(x) = \begin{cases} 2x, & x \in [0, \frac{1}{2}], \\ 2(1 - x), & x \in (\frac{1}{2}, 1]. \end{cases}$$



   (a) Find the fixed points of equation (1) and determine their stability.

   (b) Equation (1) has a solution of period two. Find the two points in this solution and determine the solution's stability. [Hint: when calculating $f(f(x))$, don't forget that $f(x)$ increases from 0 to 1 as x increases from 0 to $\frac{1}{2}$, then decreases from 1 to 0 as x increases from $\frac{1}{2}$ to 1. Thus $f(f(x))$ ought to be two tents side-by-side.]