

MTH 370, Fall 2009
Homework 1

Instructions: Do these calculations by hand (you may use a computer or calculator for simple arithmetic and function evaluations) and show your work.

1. Consider a population of cells that divides synchronously, and that at each division a cell divides into $k \geq 1$ cells. In addition, suppose that immediately after each division a fraction p of the cells die.
 - (a) Write down the first-order linear difference equation describing this cell population.
 - (b) Solve the difference equation.
 - (c) Determine for what value of p the number of cells neither grows nor decays.
2. Consider a population of cells that divides synchronously, and that at each division a cell divides into $k \geq 1$ cells. In addition, suppose that immediately after each division, a number h of the cells die.
 - (a) Write down the first-order linear difference equation describing this cell population.
 - (b) Solve the difference equation.
 - (c) Determine for what value of h the number of cells neither grows nor decays.
3. Consider the nonlinear difference equation

$$c_{n+1} = rc_n e^{-c_n}. \tag{1}$$

Here c_n is not restricted to the interval $[0, 1]$ but can be any nonnegative number.

- (a) Find the fixed points of (1). [Hint: there are two.]
- (b) Determine the stability of each fixed point.
- (c) Carefully draw a cobweb diagram for $r = 2$ and $c_0 = 0.1$ and show that your diagram agrees with answers for parts (a) and (b).