Math 299 **Review for Midterm 2**

PROBLEM 1. Negate the following statement: *For all* $\epsilon > 0$*, there is some* $\delta > 0$ *such that if* $|x-3| < \delta$*, then* $|x^2 - 9| < \epsilon$.

PROBLEM 2. Express the following using $\forall$ and $\exists$: *There is some* $b \in \mathbb{R}$ *such that the equation* $x^2 - b = 0$ *has no solution.*

PROBLEM 3. Let $n \in \mathbb{N}$.
(a) Use induction to show that exactly one element of the set $\{n, n+1, n+2, n+3\}$ is divisible by 4.

(b) Use the Division Lemma to show that exactly one element of the set $\{n, n+1, n+2, n+3\}$ is divisible by 4.

PROBLEM 4 Let $x \in \mathbb{N} = \{1, 2, \ldots\}$.
(a) Prove that $x^2 + x$ is even.

(b) Prove that $(x^2 + x)/2$ is divisible by x if and only if x is odd.

(c) Prove that $(x^2 + x)/2$ is divisible by $x + 1$ if and only if x is even.

PROBLEM 5. (Houston 26.7 (iii)) Show that if $x^2 - 3x + 2 < 0$, then $1 < x < 2$.

PROBLEM 6. (Houston 27.23 (v)) Prove that every common divisor of $a, b \in \mathbb{Z}$ is a divisor of $\gcd(a, b)$.

PROBLEM 7. (a) Calculate $\gcd(52, 221)$.

(b) Find $m, n \in \mathbb{Z}$ such that $52m + 221n = \gcd(52, 221)$.

PROBLEM 8. Let $n \in \mathbb{N}$. Prove that n is composite if and only if n has a factor a that satisfies

$$1 < a \leq \sqrt{n}.$$