Math 299 **Review for Midterm 1**

PROBLEM 1. Suppose $f : A \to B$ and $g : B \to C$ are functions.

(a) Prove the following statement.

(*) If f is surjective and g is surjective, then the composition $g \circ f : A \to C$ is surjective.

(b) Identify the hypothesis and conclusion of statement (*), above.

(c) State the necessary and sufficient conditions of statement (*).

(d) State the inverse, contrapositve and converse of statement (*).

(e) State briefly why the contrapositive of statement (*) is true. Show that the inverse and converse of statement (*) are false by providing a counterexample.

PROBLEM 2. Prove the following statements. The symbol $\cong$ denotes *bijective correspondence.*

(a) Suppose A is a set. Then $A \cong A$.

(b) For all sets A and B, if $A \cong B$, then $B \cong A$.

(c) For all sets A, B and C, if $A \cong B$ and $B \cong C$, then $A \cong C$. *Hint: Use problem 1 above, and one of the homework or essay problems.*

PROBLEM 3. Let E denote the set of even integers.

(a) Describe E using set notation, and without using English words.

(b) Use a picture to illustrate a bijection between $\mathbb{N}$ and $E \times E$.

(c) Use a picture to illustrate a bijection between $\mathbb{Z}$ and $E \times E$.

(d) Show that $E \times (-1, 1)$ is in bijective correspondence with $E \times (0, 4)$. Here $(-1, 1)$ and $(0, 4)$ are the intervals in $\mathbb{R}$.

PROBLEM 4.

(a) Find a set $S \subseteq \mathbb{R}$ such that the function

$$\begin{aligned} f : [0, \infty) &\longrightarrow S \\ x &\longmapsto \frac{1}{1+x^2} \end{aligned}$$

is surjective.

(b) Let f and S be as in part (a). Prove that f is injective.

(c) Let S be as in part (a). Suppose T is a set which is strictly larger than S; that is, $S \subseteq T$, but $S \neq T$. Explain why the function

$$\begin{aligned} g : [0, \infty) &\longrightarrow T \\ x &\longmapsto \frac{1}{1+x^2} \end{aligned}$$

is not surjective.