Math 299 **Supplement: Axioms for $(\mathbb{R}, +, \cdot)$** **Nov 15, 2013**

AXIOM 8.1. For all $x, y, z \in \mathbb{R}$,

(i) $x + y = y + x$
(ii) $(x + y) + z = x + (y + z)$
(iii) $x \cdot (y + z) = x \cdot y + x \cdot z$
(iv) $x \cdot y = y \cdot x$
(v) $(x \cdot y) \cdot z = x \cdot (y \cdot z)$

AXIOM 8.2. There exists a number 0, such that for all $x \in \mathbb{R}$, $x + 0 = x$

AXIOM 8.3. There exists a number 1 such that $1 \neq 0$ and whenever $x \in \mathbb{R}$, $x \cdot 1 = x$

AXIOM 8.4. For each $x \in \mathbb{R}$, there exists a real number, denoted by $-x$, such that $x + (-x) = 0$

AXIOM 8.5. For each $x \in \mathbb{R}\backslash\{0\}$, there exists a real number, denoted by x^{-1}, such that $x \cdot x^{-1} = 1$

AXIOM 8.26. There exists a subset $\mathbb{R}^{>0}$ of $\mathbb{R}$ satisfying

(i) If $x, y \in \mathbb{R}^{>0}$ then $x + y \in \mathbb{R}^{>0}$
(ii) If $x, y \in \mathbb{R}^{>0}$ then $x \cdot y \in \mathbb{R}^{>0}$
(iii) $0 \notin \mathbb{R}^{>0}$
(iv) For every $x \in \mathbb{R}$, we have $x \in \mathbb{R}^{>0}$ or $x = 0$ or $-x \in \mathbb{R}^{>0}$

AXIOM 8.52. (Completeness axiom). Every nonempty subset of $\mathbb{R}$ that is bounded above has a least upper bound.