Math 299 **Review for Midterm 2**

PROBLEM 1. Let $a, b, c \in \mathbb{Z}$. Prove that if $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$, then $\gcd(a, bc) = 1$.

PROBLEM 2. Recall that the Fibonacci numbers are defined by $F_1 = 1, F_2 = 1$, and

$$F_{n+1} = F_{n-1} + F_n, \qquad n \geq 2.$$

(a) Prove that for all $n \in \mathbb{N}$, $\sum_{i=1}^{n} F_i = F_{n+2} - 1$.

(b) Prove that every natural number can be written as the sum of distinct Fibonacci numbers. (This is a harder problem. Hint: use strong induction).

PROBLEM 3. Let $a, b, c, d \in \mathbb{Z}$ with a and b nonzero. Prove that if $ab \nmid cd$, then $a \nmid c$ or $b \nmid d$.

PROBLEM 4. Let x be an irrational real number. Prove that either x^2 or x^3 is irrational.