Math 299 **Homework 13 Solutions** **Fall 2013**

1. Prove that if $ab|ac$, then $b|c$, where $a, b, c \in \mathbb{Z}$ and $a \neq 0$.

Proof: If $ab|ac$, then $ac = abk$ for some $k \in \mathbb{Z}$. Since $a \neq 0$, we can divide both sides by a to get $c = bk$, which is exactly the statement that b divides c.

2. Prove that if $a|b$ and $b|c$, then $a|c$.

Proof: If $a|b$ and $b|c$, then we can write $b = ak$ and $c = bl$ for some $k, l \in \mathbb{Z}$. Then $c = bl = akl$, and so a divides c.

3. Verify that 101 is prime, but try not to do too much work. *Hint: Prove by contradiction. Argue that if this is not true, then there is a prime that divides 101, but this prime is not very big (how big?). Then rule out cases.*

Proof: For sake of contradiction, assume 101 is not prime. Then there is some $a \in \mathbb{Z}$ such that $a|101$ and $1 < a \leq \sqrt{101} < 11$. We may assume that a is prime, so $a \in \{2, 3, 5, 7\}$. However, none of these numbers divides 101:

$$101/2 = 50+1/2, \qquad 101/3 = 33+2/3, \qquad 101/5 = 20+1/5, \qquad 101/7 = 14+3/7.$$

This is a contradiction.