Math 299 Solutions to Problems 20.14 (ii), 22.10 (v) Fall 2013

20.14 (ii) We want to show that the sum of two consecutive odd numbers is a multiple of 4. Let p, q be two consecutive odd integers. This means that we can write them in the form

$$p = 2k + 1, \qquad q = 2k + 3$$

for some $k \in \mathbb{Z}$. Then

$$p + q = 2k + 1 + 2k + 3 = 4k + 4 = 4(k + 1).$$

This is obviously divisible by 4 since $k + 1 \in \mathbb{Z}$.

22.10 (v) Let $a, b, c \in \mathbb{R}$ and suppose $a = bc$. We want to show that if any two of a, b, c are non-zero, then the third is non-zero as well. We will prove this by considering cases.

Case 1: Assume a, b are non-zero. We will prove that $c \neq 0$ by contradiction. If $c = 0$, then $bc = 0$. Since $a = bc$, this contradicts the assumption that $a \neq 0$.

Case 2: Assume a, c are non-zero. This is exactly the same as Case 1, since the roles of b and c are interchangeable.

Case 3: Assume b, c are non-zero. For sake of contradiction, assume $a = 0$. Since $b \neq 0$, we can divide by it to get $0 = b/a = c$. This contradicts the assumption that $c \neq 0$.

$\square$