**Math 299** **Essay 2 Solution** **Fall 2013**

Give a careful proof of the following statement. Use proof by contradiction.

PROPOSITION: For all $n \in \mathbb{Z}$, $n^2 + 2$ is not divisible by 4.

*Proof:* For sake of contradiction, suppose there is some $n \in \mathbb{Z}$ for which $n^2 + 2$ is divisible by 4. Then we can write

$$n^2 + 2 = 4k$$

for some $k \in \mathbb{Z}$.

First consider the case where $n = 2l$, for some $l \in \mathbb{Z}$. Then

$$4k = n^2 + 2 = 4l^2 + 2,$$

and so

$$4(k - l^2) = 2.$$

Divide both sides by 2 to get $2(k - l^2) = 1$. This is impossible since 1 is not even but the left-hand side is.

Now consider the case where $n = 2l + 1$ for some $l \in \mathbb{Z}$. Then

$$4k = n^2 + 2 = 4l^2 + 4l + 3.$$

This gives

$$4(k - l^2 - l) = 3.$$

Again, this is a contradiction since 3 is not even. Q.E.D.