Meanings

- **Definition** : an explanation of the mathematical meaning of a word.
- **Theorem** : A statement that has been proven to be true.
- **Proposition** : A less important but nonetheless interesting true statement.
- **Lemma**: A true statement used in proving other true statements (that is, a less important theorem that is helpful in the proof of other results).
- **Corollary**: A true statment that is a simple deduction from a theorem or proposition.
- **Proof**: The explanation of why a statement is true.
- **Conjecture**: A statement believed to be true, but for which we have no proof. (a statement that is being proposed to be a true statement).
- **Axiom**: A basic assumption about a mathematical situation. (a statement we assume to be true).

Examples

- **Definition 6.1**: A *statement* is a sentence that is either true or false–but not both. ([H], Page 53).
- **Theorem 10.1**: $\mathbb{N}$, considered as a subset of $\mathbb{R}$, is not bounded above. ([B], Page 96).
- **Corollary 10.2** : $\mathbb{Z}$ is not bounded above. ([B], Page 96).
- **Proposition 10.4** : For each $\varepsilon > 0$, there exists $n \in \mathbb{N}$ such that $\frac{1}{n} < \varepsilon$. ([B], Page 96).
- **Lemma** : *Lemmas are considered to be less important than propositions. But the distinction between categories is rather blurred. There is no formal distinction among a lemma, a proposition, and a theorem.*
- **Axioms** : If m and n are integers, then $m + n = n + m$. (Read [B] Page 4.)
- **Conjecture**: Mathematicians are making, testing and refining conjectures as they do their research.

Group Axioms

A **Group** is a set G together with an operation $\#$, for which the following axioms are satisfied.

A_1. **Closure:** $\forall a, b \in G,\ a\#b \in G$

A_2. **Associativity:** $\forall a, b, c \in G,\ (a\#b)\#c = a\#(b\#c)$

A_3. **Identity element:** $\exists e \in G$ such that $\forall a \in G,\ a\#e = e\#a = a$

A_4. **Inverse element:** $\forall a \in G,\ \exists b \in G$ such that $a\#b = b\#a = e$

1. Is $\mathbb{N}$ with $+$ a group?

2. Is $\mathbb{Z}$ with $+$ a group?

3. Do the axioms imply that if G is a group and $a, b \in G$ then $a\#b = b\#a$?

4. Can you give an example of a group (all axioms $A_1 - A_4$ are satisfied) whose elements do not commute with each other?

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Group Theorems

Thereom: The identity element is unique.

proof:

Thereom: For every element $a \in G$ there exists a **unique** inverse.

proof:

Axiomatic system 1:

Undefined terms: *member, committee*
A_1. Every committee is a collection of at least two members.
A_2. Every member is on at least one committee.

1. Find two different models for this set of axioms.

2. Discuss how it can be made **categorical** (there is a one-to-one correspondence between the elements in the model that preserves their relationship).

Axiomatic system 2:

Definition: A line ℓ **intersects** a line m if there is a point A that lies on both ℓ and m.

Undefined terms: *point, line*
A_1. Every line is a set of at least two points.
A_2. Each two lines intersect in a unique point.
A_3. There are precisely three lines.

Find two different models for this set of axioms.

Axiomatic system 3:
Undefined terms: *point, line, lie on*

Definition: A line ℓ **passes through** points A and B if A and B lie on ℓ.

Definition: A line ℓ **intersects** a line m if there is a point A that lies on both ℓ and m.

A_1. There are exactly five points.
A_2. Exactly two points lie on each line.
A_3. At most one line passes through any two points.
A_4. There are exactly three lines.
Find two different models for this set of axioms.

Theorem: At least one pair of lines intersect.

Conjecture: There is a point that doesn't lie on any line.