1 Applications of Functions to Economics

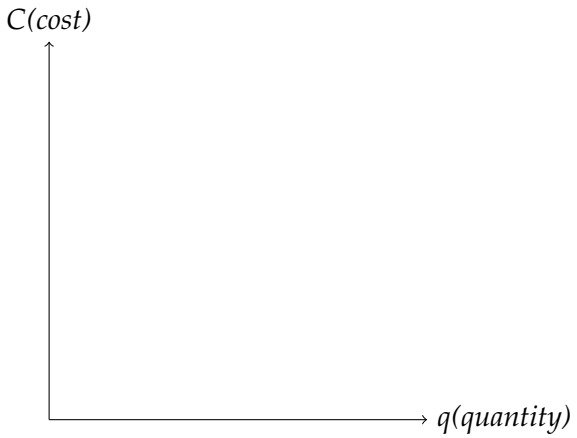
*** The Cost Function**

The **cost function**, $C(q)$, gives the total cost of producing a **quantity** q of some good.

$$\textbf{The total costs} = \textbf{Fixed Costs} + \textbf{Variable Costs},$$

where **Fixed Costs** are incurred even if nothing is produced and **Variable Costs** depend on how many units are produced.
If $C(q)$ is a linear cost function,
Fixed costs are represented by the vertical intercept.
Marginal cost is represented by the slope.

Example 1 *A company produces and sells shirts. The fixed costs are* \$7000 *and the variable costs are* \$5 *per shirt. Find a formula for the cost function* $C(q)$ *as a function of the quantity of shirts,* q*. And graph the cost function.*



*** The Revenue Function**

The **revenue function**, $R(q)$, gives the total revenue received from a firm from selling a **quantity**, q, of some good. If the good sells for a price of p per unit, then

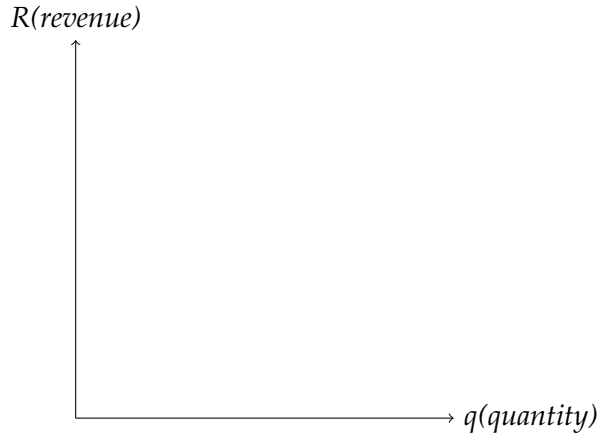
$$\textbf{Revenue} = \textbf{Price} * \textbf{Quantity},$$

which is exactly the same as

$$R = pq.$$

If the price does not depend on the quantity sold, so p is a constant, the graph of revenue as a function of q is a line through the origin, with slope equal to the price p.
The **marginal revenue** is also represented by the slope.

Example 2 *In Example 1, if the shirts are sold for* \$12 *each, find a formula for the revenue function* $R(q)$ *as a function of the quantity of shirts,* q*. And graph the revenue function.*



*** The Profit Function**

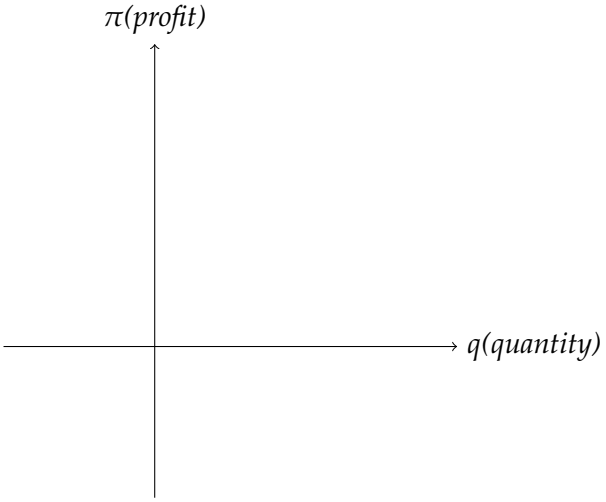
Profit $=$ **Revenue** $-$ **Cost**.

Let π denote the profit, then

$$\pi = R - C.$$

The **break-even** point is the point where the profit is zero, or equivalently, revenue equals cost.
If the profit function is a linear function, then the **marginal profit** is represented by the slope.

Example 3 *In Example 1 and Example 2, find a formula for the profit function* $\pi(q)$ *as a function of the quantity of shirts,* q*. Graph it and mark the break-even point.*



Example 4 *The following figure shows cost and revenue for a company.*

(a) Find the fixed costs and the marginal cost for the cost function $C(q)$.

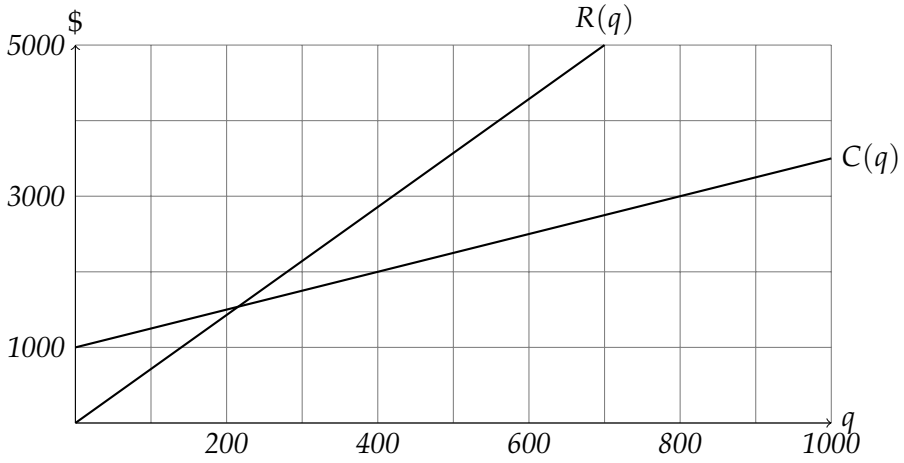
(b) Find a formula for $C(q)$.

(c) What is the marginal revenue?

(d) Find a formula for $R(q)$.

(e) Approximately what quantity does this company have to produce to make a profit?

(f) Estimate the profit generated by 500 units.



Example 5 *The following table shows a company's estimates of cost and revenue for a product.*

q	0	10	20	30	40
$C(q)$	500	600	700	800	900
$R(q)$	0	250	500	750	1000

(a) What are the fixed costs and the marginal cost?

(b) What price does the company charge for its products?

(c) Find a formula for $C(q)$ and $R(q)$.

(d) Find the break-even quantity.

Example 6 *A company that makes Adirondack chairs has fixed costs of* \$5000 *and variable costs of* \$30 *per chair. The company sells the chairs for* \$50 *each.*

(a) Find a formula for the cost function.

(b) Find a formula for the revenue function.

(c) Find the break-even point.