

Name: _____

The purpose of this worksheet is to provide an opportunity to practice differentiation formulas for section 005. It will not be graded and you are not expected to finish in class. There are commonly used formulas after the problems, some of these problems might be challenging, if you have questions, feel free to ask me after class, or come to my office during office hours.

EXERCISES

(1) $f(x) = x^2 + 2x + 1$

(13) $h(x) = \frac{(x^2+x+1)(4^x)}{x \ln x}$

(2) $g(x) = 3xe^x$

(14) $t(x) = \ln(x^2 + 3x)e^{x^2-x}$

(3) $h(x) = \ln(x^2 + x)$

(15) $n(x) = \frac{1}{\ln x+x}$

(4) $t(x) = 3x^3e^7$

(16) $a(t) = t(t^3 + t + e^t)$

(5) $n(x) = \frac{x+1}{x-1}$

(17) $f(u) = \frac{e^7+\ln 2+1}{u} - 1$

(6) $a(t) = te^{t^2}$

(18) $g(x) = \sqrt{x^3 + 2x + 1}$

(7) $f(u) = \frac{u^2}{\ln(1+e^u)}$

(19) $h(y) = e^{5y} + \ln(2y) + \frac{1}{y^5}$

(8) $g(x) = e^{\sqrt[4]{3x^4+3x^2+1}}$

(20) $s(t) = \frac{t^3+7t+1}{t}$

(9) $h(y) = \frac{1}{(7y)^2}$

(21) $f(x) = \ln(e^x(x^2 + 1))$

(10) $s(x) = \frac{(5x^3+2x^2+2)\ln(x)}{e^{3x+x}}$

(22) $g(x) = e^{(x^2+3)^7(x-1)^5}$

(11) $f(x) = (x^2 + x)^{100}$

(23) $h(x) = \ln x - \ln\left(\frac{1}{x}\right)$

(12) $g(x) = (3x^2 + x + 1)e^x \ln x$

(24) $t(x) = y(2^x - \sqrt{5})$

(25) $n(x) = (x + 1)^4(x - 1)^4x^3$

(28) $g(x) = \sqrt{\frac{x^2+x}{\ln x+1}}$

(26) $a(t) = \frac{t^2-1}{e^{t^2}-1}$

(29) $h(y) = |y^2|$

(27) $f(u) = \frac{u^4}{\ln(1+e^u)}$

(30) $s(t) = \frac{1}{|t|}$

FORMULAS

(1) $\frac{d}{dx}c = 0$

(2) $\frac{d}{dx}x^n = nx^{n-1}$

(3) $\frac{d}{dx}(mx + b) = m$

(4) $\frac{d}{dx}e^x = e^x$

(5) $\frac{d}{dx}e^{kx} = ke^{kx}$

(6) $\frac{d}{dx}a^x = a^x \ln(a)$

(7) $\frac{d}{dx} \ln(x) = \frac{1}{x}$

(8) $\frac{d}{dx}[cf(x)] = cf'(x)$

(9) $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

(10) $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$ Product Rule

(11) $\frac{d}{dx}[f(x)g(x)h(x)] = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$

$$(12) \frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \text{ Quotient Rule}$$

$$(13) \frac{d}{dx} [f \circ g(x)] = \frac{d}{dx} [f(g(x))] = f'(g(x))g'(x) \text{ Chain Rule}$$

Following formulas are special forms of formula (13), but they are most commonly used forms when you are taking the derivatives of composite functions.

$$(a) \frac{d}{dx} [e^{f(x)}] = e^{f(x)} f'(x)$$

$$(b) \frac{d}{dx} [\ln f(x)] = \frac{1}{f(x)} f'(x)$$

$$(c) \frac{d}{dx} [(f(x))^n] = n f(x)^{n-1} f'(x)$$

$$(d) \frac{d}{dx} [\sqrt{f(x)}] = \frac{1}{2} f(x)^{-\frac{1}{2}} f'(x)$$

$$(e) \frac{d}{dx} [f(g(h(x)))] = f'(g(h(x)))g'(h(x))h'(x)$$

ANSWERS

- (1) $f'(x) = 2x + 2$
- (2) $g'(x) = 3e^x + 3xe^x$
- (3) $h'(x) = \frac{1}{x^2+x}(2x + 1)$
- (4) $t'(x) = 9x^2 e^7$ Here e^7 is just a constant coefficient, has nothing to do with x , so just keep it.
- (5) $n'(x) = \frac{1 \cdot (x-1) - (x+1) \cdot 1}{(x-1)^2}$
- (6) $a'(t) = 1 \cdot e^{t^2} + t e^{t^2} (2t)$
- (7) $f'(u) = \frac{2u \ln(1+e^u) - u^2 \frac{1}{1+e^u} e^u}{(\ln(1+e^u))^2}$
- (8) $g'(x) = e^{\sqrt[4]{3x^4+3x^2+1}} \cdot \frac{1}{4}(3x^4 + 3x^2 + 1)^{-\frac{3}{4}}(12x^3 + 6x)$
- (9) $h'(y) = -2(7y)^{-3} \cdot 7$
- (10) $s'(x) = \frac{[(15x^2+4x) \ln(x) + (5x^3+2x^2+2)\frac{1}{x}](e^{3x+x}) - (5x^3+2x^2+2) \ln(x)(3e^{3x+1})}{(e^{3x+x})^2}$
- (11) $f'(x) = 100(x^2 + x)^{99}(2x + 1)$
- (12) $g'(x) = (6x + 1)e^x \ln x + (3x^2 + x + 1)e^x \ln x + (3x^2 + x + 1)e^x \frac{1}{x}$
- (13) $h'(x) = \frac{[(2x+1)4^x + (x^2+x+1) \ln 4 \cdot 4^x]x \ln x - (x^2+x+1)4^x [1 \cdot \ln x + x \frac{1}{x}]}{(x \ln x)^2}$
- (14) $t'(x) = \frac{1}{x^2+3x}(2x + 3)e^{x^2-x} + \ln(x^2 + 3x)e^{x^2-x}(2x - 1)$
- (15) $n'(x) = -(\ln x + x)^{-2}(\frac{1}{x} + 1)$
- (16) $a'(t) = 1 \cdot (t^3 + t + e^t) + t(3t^2 + 1 + e^t)$
- (17) $f'(u) = -(e^7 + \ln 2 + 1)u^{-2}$
- (18) $g'(x) = \frac{1}{2}(x^3 + 2x + 1)^{-\frac{1}{2}}(3x^2 + 2)$
- (19) $h'(y) = 5e^{5y} + \frac{1}{2y}2 - 5y^{-6}$
- (20) $s'(t) = \frac{(3t^2+7)t - (t^3+7t+1) \cdot 1}{t^2}$
- (21) $f'(x) = \frac{1}{e^x(x^2+1)}[e^x(2x) + e^x(x^2 + 1)]$ or $f'(x) = 1 + \frac{1}{x^2+1}2x$
- (22) $g'(x) = e^{(x^2+3)^7(x-1)^5}[7(x^2 + 3)^6(2x)(x - 1)^5 + (x^2 + 3)^7 5(x - 1)^4]$
- (23) $h'(x) = 2\frac{1}{x}$ can first rewrite $\ln(\frac{1}{x}) = \ln(x^{-1}) = -\ln(x)$
- (24) $t'(x) = y(\ln 2 \cdot 2^x)$ here y has nothing to do with x , so think it as a constant coefficient, just keep it.
- (25) $n'(x) = 4(x + 1)^3(x - 1)^4 x^3 + (x + 1)^4 4(x - 1)^3 x^3 + (x + 1)^4(x - 1)^4 3x^2$

$$(26) a'(t) = \frac{(2t)(e^{t^2}-1)-(t^2-1)(e^{t^2} 2t)}{(e^{t^2}-1)^2}$$

$$(27) f'(u) = \frac{4u^3 \ln(1+e^u) - u^4 \frac{1}{1+e^u} e^u}{(\ln(1+e^u))^2}$$

$$(28) g'(x) = \frac{1}{2} \left(\frac{x^2+x}{\ln x+1} \right)^{-\frac{1}{2}} \frac{(2x+1)(\ln x+1) - (x^2+x)\left(\frac{1}{x}\right)}{(\ln x+1)^2}$$

$$(29) h'(y) = \frac{y^2}{|y^2|} (2y)$$

$$(30) s'(t) = -(|t|)^{-2} \frac{t}{|t|}$$