

Evaluating a polynomial by synthetic division is the same as evaluating it using nested multiplication. For example, suppose

$$f(x) = 3x^3 + 2x^2 - 3x - 2 = x(3x^2 + 2x - 3) - 2 = x(x(3x + 2) - 3) - 2$$

So to evaluate $f(x)$ at $x = r$,

$$f(r) = r(r(3r + 2) - 3) - 2$$

Describing the expression for $f(r)$ in words, starting with the innermost nested term, multiply 3 by r and add 2. Then take that result and multiply it by r and then add -3 . Then take that result and multiply it by r and then add -2 .

One usually does the scratchwork as follows:

$$\begin{array}{r|rrrr}
 r & 3 & 2 & -3 & -2 \\
 & \nearrow \times r & \nearrow \times r & \nearrow \times r & \\
 & 3r & r(3r+2) & r(r(3r+2)-3) & \\
 \hline
 & 3 & 3r+2 & r(3r+2)-3 & r(r(3r+2)-3)-2 = f(r)
 \end{array}$$

For example, if $r = 2$,

$$\begin{array}{r|rrrr}
 2 & 3 & 2 & -3 & -2 \\
 & & 6 & 16 & 26 \\
 \hline
 & 3 & 8 & 13 & 24 = f(2)
 \end{array}$$

If $r = 1$,

$$\begin{array}{r|rrrr}
 1 & 3 & 2 & -3 & -2 \\
 & & 3 & 5 & 2 \\
 \hline
 & 3 & 5 & 2 & 0 = f(1)
 \end{array}$$

Notice also that the numbers in the bottom row to the left of the last entry give the coefficients of $q(x)$ when writing $f(x) = (x - r)q(x) + f(r)$. For example, when $r = 2$, $q(x) = 3x^2 + 8x + 13$ and $f(x) = (x - 2)(3x^2 + 8x + 13) + 24$. To see why the numbers in the bottom row give the coefficients of $q(x)$, divide $f(x) = 3x^3 + 2x^2 - 3x - 2$ by $x - 2$ using long division compare the computation to the computation doing synthetic division.