1. Let $u = \sin\left(\frac{x}{4}\right)$. Then $du = \cos\left(\frac{x}{4}\right) \cdot \frac{1}{4}\,dx$, so $\cos\left(\frac{x}{4}\right)\,dx = 4\,du$

   When $x = 0, u = 0$ and when $x = \pi$, $u = \frac{\sqrt{2}}{2}$. With the substitution the integral becomes

   $\int_0^{\sqrt{2}/2} 4\,du = 2u^2\Big|_0^{\sqrt{2}/2} = \frac{2 \cdot 2}{4} = 1.$

2. $\int_{-1}^{2} (2t^2+8)^{-1/2}\,t\,dt = \frac{1}{4} \cdot \frac{2}{1} \int_{-1}^{2} \frac{1}{2}(2t^2+8)^{-1/2}\,4t\,dt = \frac{1}{2}(2t^2+8)^{1/2}\Big|_{-1}^{2} = \frac{1}{2}(16^{1/2} - 10^{1/2})$

   $= \frac{1}{2}(4 - \sqrt{10}) = 2 - \sqrt{\frac{5}{2}}.$

3. $av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2}\int_{-1}^{1} (\sqrt{x^3+1})\,3\,x^2\,dx = \frac{1}{3}(x^3+1)^{3/2}\Big|_{-1}^{1} = \frac{1}{3}2^{3/2} = \frac{2}{3}\sqrt{2}.$

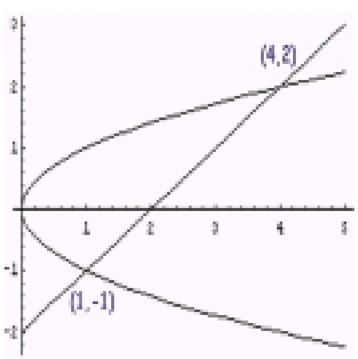
4. Suppose $F'(t) = \sqrt{1+t^2}$, i.e. $F(t)$ is an antiderivative of $\sqrt{1+t^2}$.

   Then by the Fundamental Theorem of Calculus,

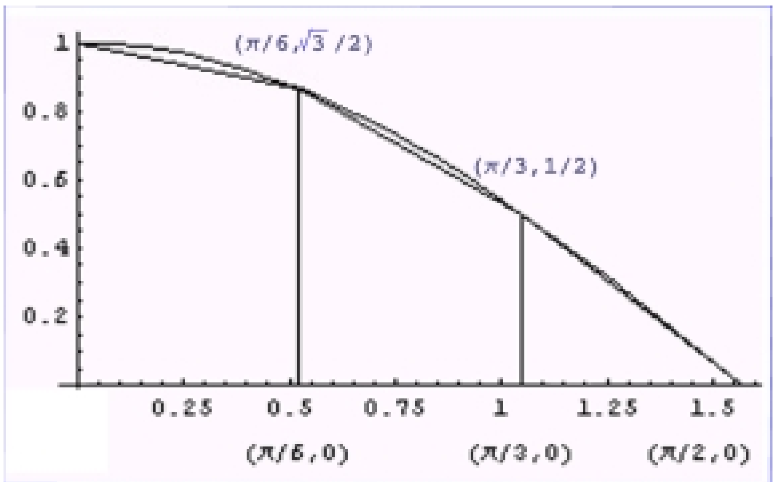
   $f(x) = F(\cos x) - F(0)$, so $f'(x) = F'(\cos x)(-\sin x) - 0 = \sqrt{1+\cos^3 x}\,(-\sin x)$.

5. The curves intersect where $y^2 = y + 2$, $y^2 - y - 2 = 0$, $(y-2)(y+1) = 0$, $y = 2$ or $y = -1$.

   Area $= \int_{-1}^{2} (y^2 + 2 - y^2)\,dy = \frac{9}{2}$



6. The trapezoidal approximation $T = T_1 + T_2 + T_3 = \frac{1}{2}(1 + \frac{\sqrt{3}}{2})\frac{\pi}{6} + \frac{1}{2}(\frac{\sqrt{3}}{2} + \frac{1}{2})\frac{\pi}{6} + \frac{1}{2}(\frac{1}{2} + 0)\frac{\pi}{6}$
$= \frac{\pi}{12}(1 + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{2} + 0) = \frac{\pi}{12}(2 + \sqrt{3}).$



7. (a) Let $f(x)$ be continuous on $[a, b]$. Suppose $F'(x) = f(x)$ on $[a, b]$. Then
$$\int_a^b f(x)\,dx = F(b) - F(a)$$

(b) Let $f(x)$ be continuous on $[a, b]$. There exists c in $[a, b]$ such that
$$f(c) = \frac{1}{(b-a)} \int_a^b f(x)\,dx$$