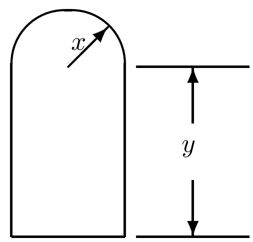
1. Use long division or synthetic division to find $\dfrac{x^2+3x+1}{x-1} = x+4+\dfrac{5}{x-1}$.

   vertical asymptote: $x = 1$.

   horizontal asymptote: none

   other asymptote: $y = x + 4$.

2. 

   $2x + 2y + \pi x = 100, \quad y = (1/2)(100 - 2x - \pi x) = 50 - (1+\pi/2)x$

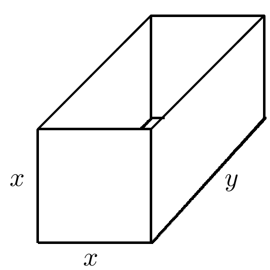
   $A = 2xy + (1/2)\pi x^2 = 2x(50 - (1+\pi/2)x) + (1/2)\pi x^2 = -(\pi/2+2)x^2 + 100x$

   $A' = -2(\pi/2+2)x + 100 = 0$ at $x = \dfrac{100}{2(\pi/2+2)} = \dfrac{100}{\pi+4}$.

   Now substitute this value of x into $y = 50 - \left(1+\dfrac{\pi}{2}\right)x$ to get $y = 50 - \left(1+\dfrac{\pi}{2}\right)\dfrac{100}{\pi+4} = \dfrac{100}{\pi+4}$

3. $x^2y = 6400$. $C = \dfrac{3}{4}xy + \dfrac{1}{2}x^2 + \dfrac{1}{2}xy = \dfrac{5}{4}xy + \dfrac{1}{2}x^2 = \dfrac{5}{4}x\left(\dfrac{6400}{x^2}\right) + \dfrac{x^2}{2} = \dfrac{x^2}{2} + \dfrac{8000}{x}$

   $C' = x - \dfrac{8000}{x^2} = 0$ at $x^3 = 8000 = 2^3 \cdot 10^3$ i.e. at $x = 20$. At this value of x, $\quad y = \dfrac{6400}{20^2} = 16$.



4. $f(x) = x^{1/3}, \quad f'(x) = \dfrac{1}{3}\,x^{-2/3}$. With $x = 64$ and $\Delta x = -1$,

   $f(x+\Delta x) \approx f(x) + f'(x)\Delta x = 64^{1/3} + \dfrac{1}{3}\,\dfrac{1}{(64)^{2/3}}\,(-1) = 4 + \dfrac{1}{3}\cdot\dfrac{1}{16}\cdot(-1) = 4 - \dfrac{1}{48} = \dfrac{191}{48}$.

5. $f(x) = x^3 + 2x - 2, \quad f'(x) = 3x^2 + 3$

$$x_1 = x_0 - \frac{f(x_0)}{x'(x_0)} = \frac{1}{2} - \frac{1/8 + 1 - 2}{3\,(1/4) + 2} = \frac{1}{2} + \frac{7/8}{3/4 + 8/4} = \frac{1}{2} + \frac{7}{8} \cdot \frac{4}{11} = \frac{1}{2} + \frac{7}{22} = \frac{18}{22} = \frac{9}{11}.$$

6. (a) $\int x^{-5/4}\,dx = -4 \cdot \int (-1/4)\,x^{-5/4}\,dx = -4x^{-1/4} + C$

(b) $\displaystyle\int \cos^2 8x\,dx = \int \frac{1}{2}\,(1 + \cos 16x)\,dx = \frac{1}{2}\,(x + \frac{\sin 16x}{16}) + C$

7. $v = 4t^2 - \cot x + C, \quad -7 = 4\left(\frac{\pi}{4}\right)^2 - 1 + C, \quad C = -6 - \frac{\pi^2}{4}$

$\therefore v = 4t^2 - \cot x - 6 - \dfrac{\pi^2}{4}.$