1. $\dfrac{\sin(x^2+x)}{x} = \dfrac{\sin(x^2+x)}{(x^2+x)} \cdot \dfrac{(x^2+x)}{x} = \dfrac{\sin(x^2+x)}{(x^2+x)} \cdot (x+1) \to 1 \cdot 1 = 1$ as $x \to 0$.

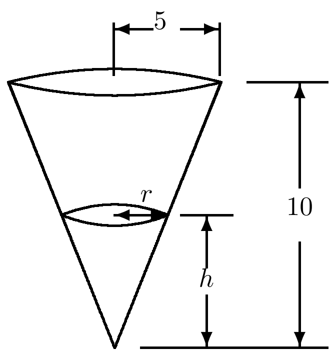
2. a) $\dfrac{d}{dx}\sin(x^2) = \cos(x^2) \cdot 2x,$ b) $\dfrac{d}{dx}\sin^2 x = 2\sin x \cos x$

3. $\dfrac{d}{dx}\sin(\cos x) = \cos(\cos x)(-\sin x)$

4. $\dfrac{r}{h} = \dfrac{5}{10}, \quad r = (1/2)h, \quad V = (1/3)\pi r^2 h = (1/3)\pi(h^2/4) = (\pi/12)h^3, \quad \dfrac{dV}{dh} = (\pi/4)h^2$

$\dfrac{dV}{dt} = \dfrac{dV}{dh}\dfrac{dh}{dt} = \dfrac{\pi}{4} \cdot h^2 \cdot \dfrac{dh}{dt}.$

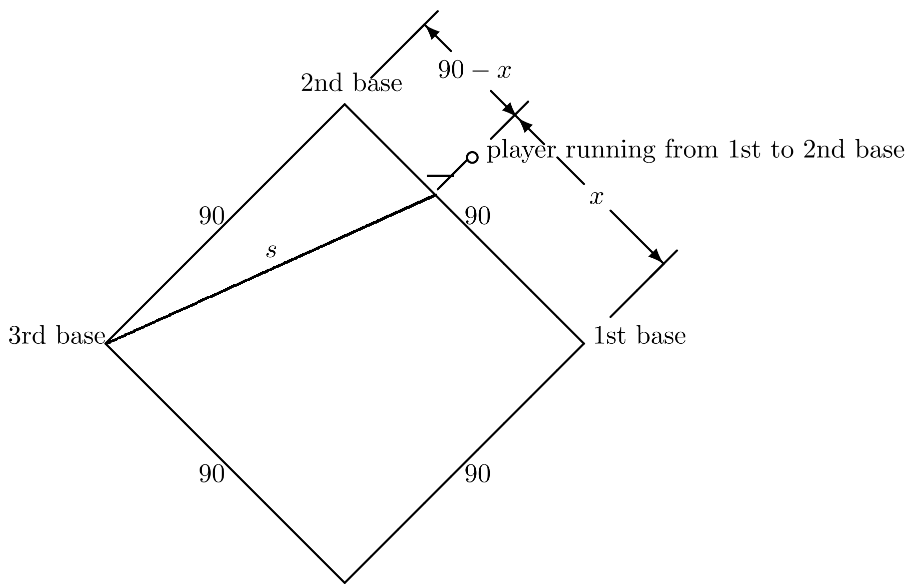
When $h = 6, \quad 9 = \dfrac{\pi}{4} \cdot 36 \cdot \dfrac{dh}{dt}$ so $\dfrac{dh}{dt} = \dfrac{1}{\pi}$ ft/min.



5. $s^2 = 90^2 + (90-x)^2$, $2s\dfrac{ds}{dt} = 2(90-x)(-\dfrac{dx}{dt})$, $\dfrac{ds}{dt} = \left(\dfrac{90-x}{s}\right)\cdot\left(-\dfrac{dx}{xt}\right)$

When $x = 30$, $s = \sqrt{90^2+60^2} = 10\sqrt{117} = 10\sqrt{9\cdot 13} = 30\sqrt{13}$.

Then $\dfrac{ds}{dt} = \left(\dfrac{90-30}{30\sqrt{13}}\right)\cdot(-16) = \dfrac{-32}{\sqrt{13}} = \dfrac{-32\sqrt{13}}{13}$



6. Differentiate $x^2 - xy + y^2 = 7$ implicitly with respect to x to get

$2x - (xy' + y) + 2y\cdot y' = 0$ Sove for y':

$y' = \dfrac{y-2x}{2y-x}$

The slope m at $(-1,2)$ is $m = \left(\dfrac{y-2x}{2y-x}\right)|_{x=-1,\ y=2} = \dfrac{2-2\cdot(-1)}{2\cdot 2-(-1)} = \dfrac{4}{5}$

The tangent line at $(-1,2)$ is the line through $(-1,2)$ with slope $m = \dfrac{4}{5}$, which is given by

$\dfrac{y-2}{x-(-1)} = \dfrac{4}{5}$, or $y = \dfrac{4}{5}x + \dfrac{14}{5}$

The normal line at $(-1,2)$ is the line through $(-1,2)$ with slope $m = -\dfrac{1}{(\frac{4}{5})} = -\frac{5}{4}$, which is given by

$\dfrac{y-2}{x-(-1)} = -\dfrac{5}{4}$, or $y = -\dfrac{5}{4}x + \dfrac{3}{4}$

7. If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b) then there exists c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

8. $\dfrac{f(b)-f(a)}{b-a} = \dfrac{8^{1/3}-0}{9-1} = \dfrac{2}{8} = \dfrac{1}{4}$

$f'(c) = \dfrac{1}{3}(c-1)^{-2/3} = \dfrac{1}{4}, \quad (c-1)^{-2/3} = \dfrac{3}{4}, \quad (c-1)^{3/2} = \dfrac{4}{3}, \quad (c-1)^2 = \left(\dfrac{4}{3}\right)^3 = \dfrac{64}{27}$

$c-1 = \sqrt{\dfrac{64}{27}} = \dfrac{8}{3\sqrt{3}}, \quad \therefore c = 1 + \dfrac{8\sqrt{3}}{9} = \dfrac{9+8\sqrt{3}}{9}$

9. $f'(x) = 4x^3 - 8x = 4x(x^2-2) = 4x(x-\sqrt{2})(x+\sqrt{2})$

$f'(x)$ is negative if $x < -\sqrt{2}$, positive if $-\sqrt{2} < x < 0$, negative if $0 < x < \sqrt{2}$, and positive if $x > \sqrt{2}$.

So $f(x)$ is increasing on $(-\sqrt{2},0)\cup(\sqrt{2},\infty)$ and decreasing on $(-\infty,-\sqrt{2})\cup(0,\sqrt{2})$.

$f''(x) = 12x^2 = 4(3x^2-2)$ is positive if $x < -\sqrt{2/3}$ or $x > \sqrt{2/3}$ and negative if $-\sqrt{2/3} < x < \sqrt{2/3}$.

So $f(x)$ is concave up on $(-\infty,-\sqrt{2/3})\cup(\sqrt{2/3},\infty)$ and concave down on $(-\sqrt{2/3},\sqrt{2/3})$

10. $2x - (xy'+y) + 2yy' = 0$. At $x=-1,\ y=1, \quad -2-(-y'+1)+2y' = 0, \quad -2+y'-1+2y' = 0, \quad 3y' = 3, \quad y' = 1$.

Differentiate again to get

$2-(xy''+y'+y')+2(y\,y'') = 0$. Substitute $x=-1,\ y=1,\ y'=1$ to get

$2-(-y''+2)+2(y''+2) = 0, \quad y'' = -\dfrac{2}{3}$