Conditional Probability Homework Problems

1. Because of air pollution, in some cities on days when the air pollution index is very high, people should not exercise outdoors. Sam lives in a city where the probability of the being to exercise outdoors on any randomly chosen day is .99. Henry lives in a city where the probability of being able to exercise outdoors is .9999 on any randomly chosen day. What is the probability that Sam can exercise outdoors every day of the year? What is the probability that Henry can exercise outdoors every day of the year?

2. In a certain city 30% of the families have no bicycle, 50% have one bicycle, 15% have two bicycles, and 5% have at more than two bicycles. Assume that all people store their bicycles in the garage.
   a) What is the probability that a family has at least two bicycles?
   b) It is observed that there is a bicycle belonging to a family in its garage. Now what is the probability that the family has at least two bicycles?

3. Two dice are tossed. What is the probability that the sum is 7, given that the first die is 3?

4. Two dice are tossed. What is the probability that the first die is 3, given that the sum is 7?

5. Two dice are tossed. What is the probability that the first die is 3, given that the sum is 5?

   Four cards are drawn from a standard deck.

6. What is the probability that they are all hearts?

7. What is the probability that the fourth card is a heart, given that the first three cards are hearts?

8. In 1988, the state of Illinois required HIV testing for a couple to obtain a marriage license. The HIV testing at the time consisted of two separate tests, the ELISA test and the Western Blot test. The Elisa test was significantly less expensive. A person who is HIV positive would test positive under the ELISA test 95% of the time. A person who is HIV negative would test positive under the ELISA test 99% of the time. In 1988, it was estimated that the percentage of people applying for a marriage license that were actually HIV positive was 1%. Calculate the conditional probability under that someone who tested positive under the ELISA test was actually HIV positive. (Hint: To do this, you may assume a population of 10000 people, calculate how many of them are HIV positive and HIV negative. For each of these, you will then want to calculate how many test positive and test negative.)

9. If someone test positive on the ELISHA test, then that person is given the Western Blot test. A person who is HIV positive will test positive on the Western Blot test 99% of the time, while a
person who is HIV negative will test positive on the Western Blot test 5% of the time. Suppose we are in the problem before. What is the conditional probability that someone is HIV positive given that the person tests positive on both the Western Blot and the ELISHA test?

10. In the 2000 Summer Olympics, the head of the Australian drug testing said that the odds of the lab giving a positive result on a negative sample for an athlete was about 1 in 1000, or about 0.1%. Suppose that the tests are 100% accurate on positive samples (so they will all test positive), and that 5% of the athletes are actually using drugs. What is the conditional probability that an athlete who tested positive for drug use, actually had a positive sample? What would be the conditional probability if 1% of the athletes had positive samples?

11. For Olympic drug testing, the best tests are used by the most careful labs. Suppose the rate of false positive tests given by a lab is about 5%, while about 0.5% of the population at large uses a given drug. Again assuming that all positive samples test positive, in this case, what is the conditional probability of a positive sample given a positive test?