Extra problems for Homework 12

Due Wednesday, 4/11/07

1. Let M and N be closed, connected, oriented manifolds of dimension n. Let $M\#N$ be their connected sum. Prove that

$$\tilde{H}_k(M\#N) \cong \tilde{H}_k(M) \oplus \tilde{H}_k(N),$$

for all $0 \le k < n$. *Hint: be careful when $k = n - 1$!*

2. Let M_g and M_h be the oriented surfaces of genus g and h, respectively.

  (a) When $g \ge h$, construct a degree–1 map $f : M_g \to M_h$.

  (b) Prove that when $g > 0$, every map from S^2 to M_g has degree 0. *Hint: first, do the case when $g = 1$.*

Extra credit. Show that when $g < h$, every map from M_g to M_h has degree 0.