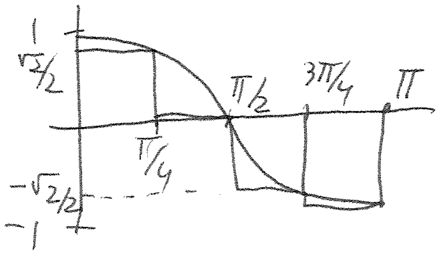
# Chapter 5 Review Problems

Math 132-06, Fall 2005

**1.** Consider the function $f(x) = \cos(x)$ on the interval $[0, \pi]$. Compute $R_4$, the right-hand estimate for $\int_0^\pi \cos(x)dx$ using 4 rectangles. Is this an over- or under-estimate?




$$R_4 = \frac{\pi}{4} \cdot \frac{\sqrt{2}}{2} + \frac{\pi}{4} \cdot 0 + \frac{\pi}{4} \cdot \frac{-\sqrt{2}}{2} + \frac{\pi}{4} \cdot -1$$
$$= \frac{-\pi}{4}.$$

This is an <u>under-estimate</u>. $f(x)$ is decreasing, so the rectangles are below the function. (This is true even when $f(x)$ is negative.)

Another way to see this is to compute that $\int_0^\pi \cos(x)\,dx = 0$, so a negative estimate is an ~~over~~ under-estimate.

**2.** Compute the following antiderivatives. *Hint: you can always take the derivative of your answer to check that you did the problem correctly.*

(a) $\int 4x^{1/3} - 2\sec(x)\tan(x)\,dx$ $= 4 \cdot \frac{3}{4} x^{4/3} - 2\sec(x) + C$

Don't forget the $+C$!

(b) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$ $= \int \cos u \cdot 2\,du = 2\sin u + C$
$= 2\sin(\sqrt{x}) + C$

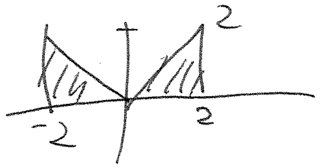
$u = \sqrt{x}$

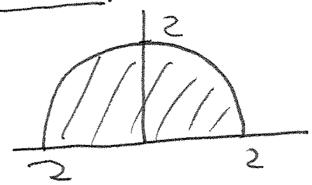
$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

**3.** Compute the following definite integrals.

(a) $\int_{-2}^{+2} |x| + \sqrt{4-x^2}\, dx.$ $\boxed{= 4 + 2\pi}$ *Hint: break up the integral and use geometry.*




$\int_{-2}^{2} |x|\, dx = 4$ $\qquad$ $\int_{-2}^{2} \sqrt{4-x^2}\, dx = \frac{1}{2} \cdot \pi (2)^2 = 2\pi$

(b) $\int_{-\pi/2}^{\pi/2} \cos^4(x) \sin(x)\, dx \;=\; \int_0^0 -u^4\, du = 0.$

$u = \cos(x)$

$du = -\sin(x)\, dx$

$x = -\frac{\pi}{2} \to u = \cos(-\frac{\pi}{2}) = 0$

$x = \pi/2 \to u = \cos(\frac{\pi}{2}) = 0$

**4.** Compute the derivatives of the following functions.

(a) $f(x) = \int_0^x \frac{1}{t^2+1}\, dt$ $\qquad$ $f'(x) = \frac{1}{x^2+1}$

(by F.T.C., part I)

(b) $g(x) = \int_x^1 \frac{1}{t^2+1}\, dt \;=\; -\int_1^x \frac{1}{t^2+1}\, dt$

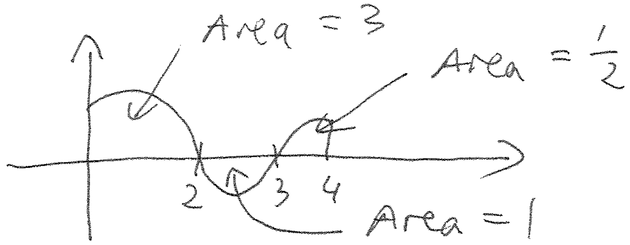
$g'(x) = \frac{-1}{x^2+1}$

(c) $h(x) = \int_x^{x^2} \frac{1}{t^2+1}\, dt \;=\; \int_x^0 \frac{1}{t^2+1}\, dt + \int_0^{x^2} \frac{1}{t^2+1}\, dt$

$h'(x) = \frac{1}{x^2+1} + \frac{1}{(x^2)^2+1} \cdot 2x$

Chain rule!

**5.** Below is a graph of $f(x)$.




What is $\int_0^4 f(x)\,dx$ ? What is $\int_0^4 |f(x)|\,dx$ ?

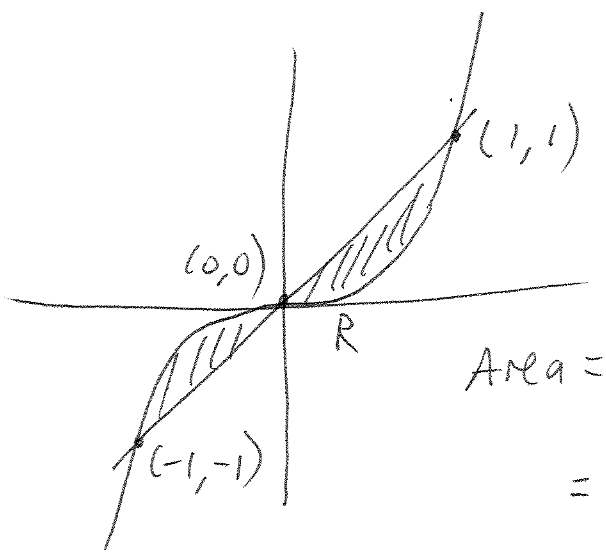
$$\int_0^4 f(x)\,dx = 3 - 1 + \frac{1}{2} = 2\tfrac{1}{2}$$

$$\int_0^4 |f(x)|\,dx = 3 + 1 + \frac{1}{2} = 4\tfrac{1}{2}$$

**6.** Let $R$ be the region enclosed between the graphs of $y = x$ and $y = x^3$. What is its area? *Hint: make sure to find all the intersection points of the two curves, and graph the region.*

Intersection points:

$$x^3 = x$$
$$x^3 - x = 0$$
$$x(x+1)(x-1) = 0$$
$$x = 0,\ x = -1,\ x = 1.$$




Notice different order! Bigger function on top.

$$\text{Area} = \int_{-1}^{0} x^3 - x\,dx + \int_0^1 x - x^3\,dx$$

$$= \left.\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left.\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$$

$$= 0 - \frac{1}{4} - \left(-\frac{1}{2}\right) + \frac{1}{2} - \frac{1}{4} - 0$$

$$= \frac{1}{2} - \frac{1}{4} + \frac{1}{2} - \frac{1}{4}$$

$$= \tfrac{1}{2}.$$