Chapter 3 Review Problems

Math 132-06, Fall 2005

1. Let $f(x) = \dfrac{3}{2x-1}$. Calculate $f'(x)$ using the limit definition of the derivative. (*Note that you can use the derivative rules to check your answer – but you must use the limit definition to get credit on the test.*)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{3}{2(x+h)-1} - \frac{3}{2x-1}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{3(2x-1) - 3(2(x+h)-1)}{(2(x+h)-1)(2x-1)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{\cancel{6x}-\cancel{3}-\cancel{6x}-6h+\cancel{3}}{(2(x+h)-1)(2x-1)}\right)$$

$$= \lim_{h\to 0} \frac{-6}{(2(x+h)-1)(2x-1)}$$

$$= \frac{-6}{(2x-1)^2}.$$

2. Suppose that x and y are related by the equation $xy = \cot(xy)$. Solve for $\frac{dy}{dx}$.

$$y + x\frac{dy}{dx} = -\csc^2(xy)\left(y + x\frac{dy}{dx}\right)$$

$$y + x\frac{dy}{dx} = -y\csc^2(xy) + -x\frac{dy}{dx}\csc^2(xy)$$

$$x\frac{dy}{dx} + x\frac{dy}{dx}\csc^2(xy) = -y - y\csc^2(xy)$$

$$\frac{dy}{dx} = \frac{-y - y\csc^2(xy)}{x + x\csc^2(xy)}$$

3. Use linearization to estimate $(1.002)^{50}$. (*What is your function $f(x)$, and at what point is it easy to evaluate?*)

$$f(x) = x^{50} \qquad f(1) = 1$$

$$f'(x) = 50x^{49} \qquad f'(1) = 50 \cdot 1^{49} = 50$$

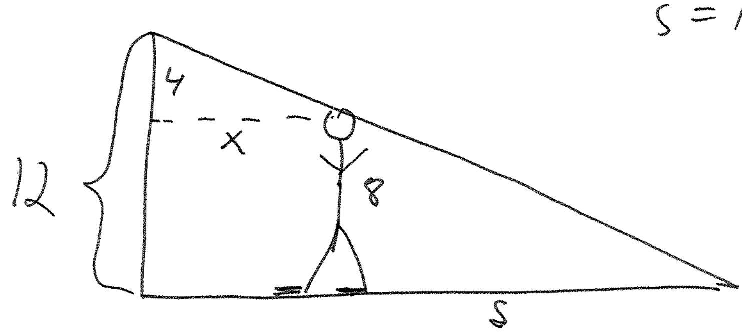
$$L(x) = 1 + 50(x-1)$$

$$L(1.002) = 1 + 50(0.002)$$

$$= 1.1.$$

4. Bigfoot is 8' tall, and is wandering the streets of East Lansing. He is walking at 2 ft/sec toward a streetlight that is 12' from the ground. How fast is the length of his shadow changing when he is 6 feet from the lamppost?

x = distance
s = length of shadow



$$\frac{x}{4} = \frac{s}{8}$$

$$8x = 4s$$

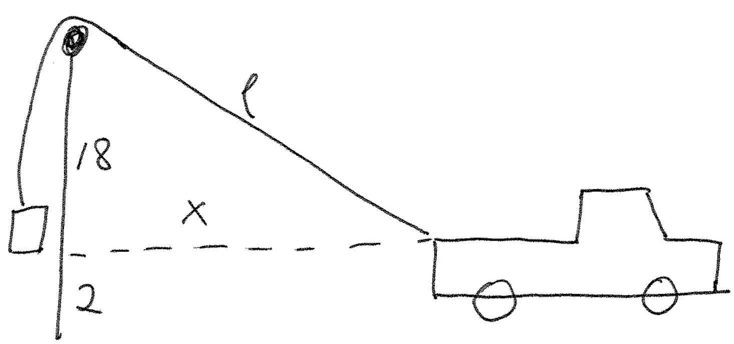
$$2x = s$$

$$2\frac{dx}{dt} = \frac{ds}{dt}.$$

$\frac{dx}{dt} = -2$ ft/sec (toward the light)

$\frac{ds}{dt} = 2(-2) = -4 \frac{\text{ft}}{\text{sec}}.$

5. A weight is attached to a 50' rope that runs over a pulley 20' above the ground, and is then attached to a truck at height 2' above the ground. The truck drives away at a speed of 9 ft/sec. How fast is the weight rising when it is 6' above the ground? (*Draw a picture, and choose your right triangle carefully!*)



$$18^2 + x^2 = \ell^2$$

$$2x\frac{dx}{dt} = 2\ell\frac{d\ell}{dt}$$

$$x\frac{dx}{dt} = \ell\frac{d\ell}{dt}$$

$$18\sqrt{3}\cdot 9 = 36\frac{d\ell}{dt}$$

$$\frac{d\ell}{dt} = \frac{9\sqrt{3}}{2} \text{ ft/sec.}$$

At this moment: $\ell = 36'$.