

Exploratory Problem: Cardiac Output

Math 42, Fall 2004

Suppose you are a doctor who wants to measure the cardiac output of a patient's heart – i.e. the amount of blood that the heart pumps per unit time. One way of doing this is the *dye dilution method*. It involves injecting some dye into an atrium of the heart, and measuring the concentration of dye that flows out through the aorta. Suppose that you inject 5 mg of dye. You measure that at time t , in seconds after you inject the dye, the concentration flowing out is

$$c(t) = t^2 e^{-t/3} \text{ milligrams per liter.}$$

What you don't know is the rate of flow of blood, F , in liters per second. But you will find it!

1. In the time interval from t seconds to $t + \Delta t$ seconds, find the quantity of dye (in milligrams) that flowed through the aorta, in terms of F and the function $c(t)$, which you know.

2. Suppose you are taking readings in 1-second intervals. In terms of your answer for part (a), set up a sum that approximates the total quantity of dye that flowed through the patient's aorta in the first 10 seconds.

3. Set up an integral expressing the exact amount of dye that flowed through the patient's aorta from time 0 to time T .

4. How much dye will flow through until time ∞ ?

5. Remember, you know that you injected 5 mg of dye. Use this information to find the rate of flow, F .