Worksheet on Dimension

Math 51, Winter 2005

The point of this worksheet is to explore what we mean by the concept of *dimension*, and to come up with a reasonable definition of it.

Each of the following questions describes a set S. Decide what you think the dimension of S is, and *why*. The questions start easy and get harder as they go along, so work together with your neighbors, and do as many as you can.

1. S is a line in $\mathbb{R}^2$.

2. S is a line in $\mathbb{R}^3$.

3. S is a knotted curve in $\mathbb{R}^3$.

4. S is a paraboloid in $\mathbb{R}^3$, given by the equation $z = x^2 + y^2$.

5. A unicyclist is showing off his skills in White Plaza. S is the set of all alignments of his unicycle, describing both where he is and which way he is facing.

6. The Hubble Space Telescope is orbiting around the earth. Assume that it can go anywhere in near-earth orbit (i.e. is not tied to any given radius from the earth's center), and can point in the direction of any star it wants to explore. S is the set of all its alignments.

7. Math 51 has eight homework assignments, two midterms, and one final. S describes all the ways that a student can get a course average of 90%.

8. What is the unifying idea behind how you determined the dimensions of these examples? Can you come up with a definition that will describe the dimension of an abstract set?