MTH 411: Midterm exam 2
Fall 2016

Duration: 50 min
No calculator allowed

Exercise 1:

1) Show that the intersection of the subgroups $\langle 3 \rangle$ and $\langle 15 \rangle$ of U_{28} is $\{1\}$.

2) Deduce from this that $U_{28} \simeq \mathbb{Z}_6 \times \mathbb{Z}_2$.

Exercise 2:

Let G be a group of order 380. We assume, by contradiction, that G is simple.
1) Compute the number of 5 and 19 Sylow subgroups of G.

2) Show that G must contain at least 304 elements of order 5 and at least 360 elements of order 19. Conclude.

Exercise 3:

From Exam 1, you remember that the set $G = \{(a,b) \in \mathbb{R}^* \times \mathbb{R}\}$ is a group for the operation
$$(a,b)(c,d) = (ac, ad + b)$$
and that $N = \{(1,x) \text{ for } x \in \mathbb{R}^*\}$ is a normal subgroup of G.

1) Find the center Z of G.

2) Using the map φ :
$$G \to \mathbb{R}^*$$
$$(a,b) \mapsto a$$
show that $G/N \simeq (\mathbb{R}^*, \times)$

Exercise 4:

In $\mathbb{Z}[i]$, what is the gcd of $3 - 3i$ and $1 + 5i$?