MTH 411: Midterm exam 1
Fall 2016

Duration: 50 min
No calculator allowed

Exercise 1:

1) Find all elements in the cyclic subgroup $\langle 4 \rangle$ generated by 4 in U_{17}.
What is the index of $\langle 4 \rangle$ in U_{17}?

2) Find all elements in the cyclic subgroup $\langle 10 \rangle$ generated by 10 in $\mathbb{Z}_{15}$.
Show that $\mathbb{Z}_{15}/\langle 10 \rangle \simeq \mathbb{Z}_5$

Exercise 2:

Let G be the set $\mathbb{R}^* \times \mathbb{R}$ and $\cdot$ be the operation on G defined by

$$(a, b) \cdot (c, d) = (ac, ad + b)$$

1) Show that $(G, *)$ is a group with identity element $(1, 0)$.

2) Let $H = \{(1, x) \ / \ x \in \mathbb{R}\}$.
Show that H is a normal subgroup of G.

Exercise 3:

Let G be a group of finite order and H and K be two subgroups of G.

1) Show that the intersection $H \cap K$ is a subgroup of G.

2) Using Lagrange's theorem, show that $|H \cap K|$ is a common divisor of $|H|$ and $|K|$.

Exercise 4:

Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 8 & 1 & 6 & 7 & 5 & 4 \end{pmatrix}$
Is σ^{411} even or odd?