MA 16020 Quiz 4 (Lessons 6-7)

1. Find the general solution to the differential equation
$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3xe^{-y} \quad .$$

2. Peter made tea and he let it cool down in his room. The temperature in the room is $25^\circ\, C$. After 10 mintes, the tea cooled from the inital temperature $100^\circ\, C$ to $60^\circ\, C$. If Peter wants to start drinking the tea when its temperature is $45^\circ\, C$, *how much longer* does he have to wait? Round the answer (in minut3es) to two decimal places.

Solution:

1. $$e^y \frac{\mathrm{d}y}{\mathrm{d}x} = 3x \quad \Rightarrow \quad \int e^y \mathrm{d}y = \int 3x \mathrm{d}x \quad \Rightarrow \quad e^y = \frac{3}{2}x^2 + C \quad \Rightarrow$$
$$\Rightarrow \quad \underline{y = \ln\left(\frac{3}{2}x^2 + C\right)} \, .$$

2. $T(t) =$ temperature after t minutes
$$\frac{\mathrm{d}T}{\mathrm{d}t} = -k(T-25), \quad T(0) = 100,\ T(10) = 60$$

First we solve the equation:
$$\frac{1}{T-25} \cdot \frac{\mathrm{d}T}{\mathrm{d}t} = -k \quad \Rightarrow \quad \int \frac{\mathrm{d}T}{T-25} = \int (-k)\mathrm{d}t \quad \Rightarrow$$
$$\ln|T-25| = -kt + C \quad \Rightarrow \quad |T-25| = e^{-kt}e^C \quad \Rightarrow$$
$$T - 25 = \pm e^C e^{-kt} \quad \Rightarrow \quad \underline{T = 25 + A \cdot B^t} \quad (A = \pm e^C, B = e^{-k}).$$

To determine A, B, one uses the initial conditions:
$$T(0) = 100 \quad \Rightarrow \quad 100 = 25 + A \cdot B^0 \quad \Rightarrow \quad \underline{A = 75}$$
$$T(10) = 60 \quad \Rightarrow \quad 60 = 25 + 75 \cdot B^{10} \quad \Rightarrow \quad B = \left(\frac{60-25}{75}\right)^{1/10} = \underline{\left(\frac{7}{15}\right)^{1/10}}$$

Therefore, $T(t) = 25 + 75 \cdot \left(\frac{7}{15}\right)^{t/10}$. Solving $T(t) = 45$ yields
$$t/10 = \frac{\ln(20/75)}{\ln(7/15)}, \quad \text{hence} \quad t = 10 \cdot \frac{\ln(20/75)}{\ln(7/15)} \approx 17.34\, .$$

Thus, Peter has to wait $7.34 (= 17.34 - 10)$ **more minutes.**