Write your name, section number (054 for 11:30, 039 for 12:30), and quiz number on the top of your quiz. (You will need a one-line calculator.)

Place your quiz face down on your desk when you are done.

QUIZ 3

1. Find the function $g(x)$ passing through the point $(2\pi, 1)$ and whose slope at each (x, y) with $y = g(x)$ is given below by:
$$g'(x) = \frac{1 - \cos(x)}{x - \sin(x)}$$
Leave any numerical values as exact; **do NOT round anything**.
2. Find the average value of $f(x) = 5\cos(2x)$ over $[\pi/4, 3\pi/4]$. Round to 3 decimal places.

QUIZ 3 Solutions

1. First integrate $g'(x)$. Let $u = x - \sin(x)$ (so that $du = 1 - \cos(x)dx$ and hence $dx = du/(1 - \cos(x))$) to get:
$$\int \frac{1 - \cos(x)}{x - \sin(x)} dx = \int \frac{1}{u} du = \ln(|u|) + C = \ln(|x - \sin(x)|) + C$$
Letting $x = 2\pi$ yields that $C = 1 - \ln(2\pi)$ so that:
$$g(x) = \ln(|x - \sin(x)|) + 1 - \ln(2\pi)$$
2. Letting $u = 2x$ and making needed changes to integral yields:
$$\begin{aligned} f_{AVG} = \frac{\int_{\pi/4}^{3\pi/4} 5\cos(2x)dx}{\frac{3\pi}{4} - \frac{\pi}{4}} &= \frac{2}{\pi} \int_{\pi/2}^{3\pi/2} \frac{5}{2}\cos(u)du \\ &= \left[\frac{5}{\pi}\sin(u)\right]_{\pi/2}^{3\pi/2} \\ &= \frac{2}{\pi}\left(\frac{5}{2}\sin(3\pi/2) - \frac{5}{2}\sin(\pi/2)\right) \\ &= \frac{2}{\pi}\left(-\frac{5}{2} - \frac{5}{2}\right) = \frac{-10}{\pi} \approx -3.183 \end{aligned}$$