MA 16020 Quiz 11 (Lessons 24-26)

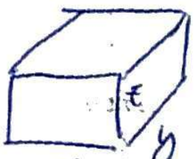
Write your name, section number (054 for 11:30, 039 for 12:30), and quiz number on the top of your quiz, **front and back.**

You may use a one-line calculator.

1. A rectangular box of volume 36 ft^3 is to be made. The cost of material is: 15 dollars per ft^2 for the bottom, 12 dollars per ft^2 for the top, and 3 dollars per ft^2 for the sides. Find the smallest possible cost of the box.

2. Find the point (x, y) where the function $f(x, y) = 3x^2 + 4xy + 6x - 15$ attains maximal value, subject to the constraint $x + y = 10$.

$\boxed{1}$



Volume $= xyz = 36 \leadsto \boxed{z = \frac{36}{xy}}$

Cost $= 15xy + 12xy + 3\cdot(2\cdot xz + 2\cdot yz) = 27xy + 6\left(x\cdot\frac{36}{xy} + y\cdot\frac{36}{xy}\right)$

$C(x,y) = 27xy + \frac{216}{y} + \frac{216}{x}$ $\quad \frac{\partial C}{\partial x} = 27y - \frac{216}{x^2}, \quad \frac{\partial C}{\partial y} = 27x - \frac{216}{y^2}$

1) $27y - \frac{216}{x^2} = 0, \quad 27x - \frac{216}{y^2} = 0$

$y = \frac{1}{27}\cdot\frac{216}{x^2} = \frac{8}{x^2}, \quad x = \frac{1}{27}\cdot\frac{216}{y^2} = \frac{8}{y^2}$ $\leadsto x = \frac{8}{(8/x^2)^2} = \frac{1}{8}x^4$

$8x - x^4 = 0$

$x(8 - x^3) = 0 \leadsto$ $x = 0$... ignore

or $8 = x^3$

$x = 2$, then $y = \frac{8}{x^2} = 2$

$\Rightarrow x = 2, y = 2$, cost $= 27\cdot 4 + \frac{216}{2} + \frac{216}{2} = 108 + 216 = 324$

$\boxed{2.}$

$\boxed{\begin{aligned} 6x + 4y + 6 &= \lambda \\ 4x &= \lambda \\ x + y &= 10 \end{aligned}}$ $\Big\}$ $6x + 4y + 6 = 4x$

$2x + 4y + 6 = 0$

$x = -2y - 3$

$(-2y - 3) + y = 10$

$-y - 3 = 10$

$y = -13, \quad x = -2(-13) - 3 = 23$