

MA 16020 Lesson 9: First-order linear differential equations I

Definition. A **first-order linear differential equation** is a differential equation that can be brought to the form:

Example 1. The differential equation

$$y' + 5y = e^{2x}$$

is first-order linear. Let's find its general solution:

The employed procedure is called **the method of integrating factors**.

Suppose that we want to solve the differential equation $y' + P(x)y = Q(x)$.

Key step: Find a function $u(x)$ (“integrating factor”) such that

Such a function can be computed as:

(we can check that this works:

)

Method of integrating factors – summary.

1. If necessary, bring the equation to the form .
2. Compute $u(x) =$.
3. Then proceed solving the equation as follows:

Exercise 2 (*Ex. 4 from last time*). A 800-gallon tank initially contains 600 gallons of pure water. Brine containing 2 pounds of salt per gallon flows into the tank at the rate of 3 gallons per minute, and the well-stirred mixture flows out of the tank at the rate of 2 gallons per minute. What is the amount of salt in the tank after 10 minutes?

Exercise 3. Find the general solution to the diff. equation

$$y' - 4\cot(4x)y = 5\sin(4x)$$

on the interval $(0, \pi/4)$.

Exercise 4. Given a function satisfying the equation

$$t^2 y' - ty = 3t^2$$

and such that $y(1) = 3$, find $y(5)$.