# MA 16020 Lesson 34: Determinants

The **determinant** of a square matrix $A$ is a certain number assigned to the matrix, $\det(A)$.

**Important properties:**

1) (multiplicativity)

2) (det of unit matrix)

3) (det and invertibility)

**Determinant of a** $1 \times 1$ **matrix** is "itself":

**Determinant of a** $2 \times 2$ **matrix** is computed as follows:

**Examples:**

$$\begin{vmatrix} 3 & 1 \\ -5 & 2 \end{vmatrix} =$$

$$\begin{vmatrix} -3 & -4 \\ 2 & 3 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 4 \\ -2 & -8 \end{vmatrix} =$$

**Determinants of $3 \times 3$ matrices:**
(A) **directly:**

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} =$$

**Example:**

$$\begin{vmatrix} 1 & 3 & 2 \\ 4 & -2 & -1 \\ 6 & 3 & 2 \end{vmatrix} =$$

(B) **using cofactors** (also works for $n \times n$ matrix, for any $n$):

For a given square matrix $A$,

its **minor** $M_{ij}$ is:

and its **cofactor** $C_{ij}$ is:

**Example:** For the matrix $\begin{bmatrix} 3 & 1 & 0 \\ -2 & 1 & 5 \\ 4 & 7 & -3 \end{bmatrix}$, one has:

the minor $M_{12} =$

the cofactor $C_{12} =$

Using cofactors, the determinant of $A$ is given as:

**Example:** Compute the determinant of the matrix $\begin{bmatrix} 3 & 1 & 0 \\ -2 & 1 & 5 \\ 4 & 7 & -3 \end{bmatrix}$ by cofactor expansion with respect to the first row.

**Inverse matrix via cofactors.** The **cofactor matrix** $C(A)$ is a matrix that has on position $(i, j)$ the cofactor $C_{ij}$.

The **adjugate matrix** $\mathrm{Adj}(A)$ is the "cofactor matrix transposed": on position $(i, j)$, it has the cofactor $C_{ji}$.

If the matrix $A$ is invertible, its inverse can be then computed by the formula:

$$A^{-1} =$$

**A particular case: $2 \times 2$ matrices.** Given an invertible matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, its inverse matrix can be found as follows:

**Example:** If it exists, find the inverse of the matrix $\begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$.