MA 16020 Lesson 33: Inverse matrices

Recall (Unit matrix): The matrix I (or, more precisely, I_n) denotes the $n \times n$ **identity matrix**

$$I_n =$$

It has the properties that 1) for every $n \times k$ matrix B,

2) for every $k \times n$ matrix C,

Inverse matrices. Given a square $(n \times n)$ matrix A, its *inverse matrix*, is an $(n \times n)$ matrix A^{-1} such that:

Example:

Matrix multiplication and systems of equations. Using matrix multiplication, a system of linear equations can be expressed as a single matrix equation.

Example:

The system
$$\begin{aligned} 2x + y + z &= 1 \\ x + 2z &= -2 \\ y - z &= 4 \end{aligned}$$
corresponds to

The inverse matrix in this case is:

We can use the inverse matrix to solve the matrix equation as follows:

How to compute the inverse matrix. Matrices for which the inverse exist are called **invertible**. If a marix is not inveritible, then it is called **singular**.

Example: If it exists, find the inverse of the matrix $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$.

Finding the inverse of a matrix – summary.

1. For the matrix A to be inverted, write down the augmented matrix:

2. Perform Gauss–Jordan elimination.

3. The matrix A^{-1} can be read off from the result as:

Exercise 1. If it exists, find the inverse of the matrix:

$$\begin{bmatrix} -3 & 0 & -9 \\ 2 & 1 & 6 \\ 1 & -2 & 4 \end{bmatrix}$$

Exercise 2. Using the inverse of the coefficient matrix, solve the following system of equations:

$$\begin{aligned} 3x - 6y + z &= 2 \\ 2y + 6z &= 3 \\ 2x - 4y + z &= 5 \end{aligned}$$

Exercise 3. If it exists, find the inverse of the matrix:

$$\begin{bmatrix} 3 & 1 & 5 \\ 1 & 0 & 1 \\ 2 & 1 & 4 \end{bmatrix}$$