MA 16020 Lesson 32: Matrix operations

Matrices. An $m \times n$ matrix is:

Examples:

Just as with numbers, there are certain operations for matrices:

Matrix addition (and subtraction). We can add and subtract matrices as long as the dimensions of the matrices agree.
The addition/subtraction is done "component-wise":

Examples:

$$\begin{bmatrix} 6 & -3 & 2 \\ -2 & 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 0 \\ 4 & -5 & 7 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 8 \\ -3 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 4 & -5 \end{bmatrix} =$$

$$\begin{bmatrix} 2 & -10 \\ -5 & 7 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 1 & 0 \\ 3 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 4 \\ 7 & -3 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} =$$

Properties of matrix addition. If A, B, C are three $m \times n$ matrices and 0 denotes the $m \times n$ matrix consisting of all zeroes, then:

1. (associativity)

2. (commutativity)

3. (neutral element)

4. ("opposite element")

Scalar multiplication of matrices. We can multiply a matrix of arbitrary dimensions by a number ("scalar"). This is done "component-wise":

Examples:

$$3\begin{bmatrix}5 & -3 & 1\\ 2 & 4 & 9\end{bmatrix} =$$

$$(-1)\begin{bmatrix}1 & 1\\ -3 & -2\end{bmatrix} =$$

$$1\begin{bmatrix}0 & 5\\ -2 & -3\\ 0 & 4\end{bmatrix} =$$

Properties of scalar multiplication. If A, B are two $m \times n$ matrices and c, d two numbers, then:

1. (associativity)
2. (distributivity)
3. (unit element)

Examples:

$$2\begin{bmatrix}1 & -5 & 3\\ 7 & 1 & 4\end{bmatrix} - 3\begin{bmatrix}2 & 0 & 3\\ 1 & 1 & -4\end{bmatrix} =$$

$$2\begin{bmatrix}1 & 0 & 2\\ 4 & 1 & 0\\ 3 & 2 & 0\end{bmatrix} + 3\begin{bmatrix}0 & 0 & 1\\ 5 & 1 & -2\\ 1 & 1 & 1\end{bmatrix} =$$

$$5\begin{bmatrix}2 & 0 & 7\\ -4 & 3 & 2\\ 1 & 5 & -2\end{bmatrix} + 4\begin{bmatrix}-3 & 4\\ 2 & -1\end{bmatrix} =$$

Matrix multiplication. If A is an $m \times n$ matrix and B is an $n \times k$ matrix (so **number of columns of** A**=number of rows of** B), we define the matrix product AB. It is an $m \times k$ matrix, whose

entry on position $(i, j) =$

Examples:

$$\begin{bmatrix} 2 & -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & -2 \\ 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix} =$$

$$\begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 0 & -3 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 0 & -3 \end{bmatrix} =$$

Properties of matrix multiplication. If A, B, C are matrices of suitable dimensions (i.e. so that the discussed operations are defined), and d a real number, then:

1. (associativity)

2. (distributivity)

3. (unit element)

4. (mult. by scalars)

!! We do not have commutativity:

Exercise. The number of grams of protein and carbohydrates per can of pet food is given by the following table:

	Protein	Carb.
Brand A	15	150
Brand B	13	140
Brand C	14	180

If we mix a meal using one can of brand A, two cans of brand B and a half can of brand C, what will be the overall amount of protein and carohydrates, respectively?