MA 16020 Lesson 31: Gauss–Jordan elimination

Recall (Gaussian elimination): Given a system of linear equations, such as

$$3x + 2y = 1,$$
$$x + y = 1,$$

we write down its associated *augmented matrix*:

Then we use the following types of row operations to obtain a matrix in the *row echelon form*:

Finally, we rewrite the matrix back as equations to determine the solution:

Gauss-Jordan elimination. Instead of the last step, we could have continued with one more row operation, to get a matrix in the *reduced row echelon form*:

reduced row echelon form =

Upshot: The solution to the system can be read off easily from the matrix:

Summary (Gauss–Jordan elimination):

Exercise 1. Find all solutions to the following system of equations:

$$2x + 5y + 4z = 3,$$
$$2x + 6y + 6z = 2,$$
$$3x + 10y + 11z = 2.$$

Exercise 2. Find the reduced row echelon form of the following augmented matrix:

$$\left[\begin{array}{ccc|c} 6 & 11 & 2 & 2 \\ 3 & 7 & 1 & -2 \\ 3 & 9 & 0 & -3 \end{array}\right]$$

Exercise 3. Find all solutions to the following system of equations:

$$2x + 2y + 2z = 5,$$
$$3x + y + 5z = 13,$$
$$x \qquad + 2z = 4.$$