MA 16020 Lesson 30: Systems of linear equations

A *linear equation* in two (three) variables is an equation of the form:

We will typically consider *systems* of linear equations.

Example. Find a solution (x, y, z) to the system of linear equations

$$
\begin{aligned}
3x + 2y + 3z &= 7, \\
4x - 3y - z &= -2, \\
x + y + z &= 3.
\end{aligned}
$$

Solution 1 (”standard”; sketch):

Solution 2 (”elimination method”):

We classify a system of linear equations based on its solutions as follows:

(A) Consistent independent:

Example:

(B) Consistent dependent:

Example:

(C) Inonsistent:

Example:

Example. Solve the system of linear equations

$$\begin{aligned} 4x + 4y + 2z &= 2, \\ 3x - 2y + z &= 0, \\ x + 4y + z &= 1. \end{aligned}$$

To make the work with the equation more efficient, we record all the relevant coefficients in the *augmented matrix* for the system:

The relevant operations for the elimination method become the following *row operations* on the matrix:

Using the row operations, we perform *Gaussian elimination*: the goal is to obtain a matrix of the form(s) (called *row echelon form*):

Let us now solve the problem using Gaussian elimination:

Exercise (if time permits). The dog nutrition from brand A contains 15 g of protein and 210 g of carbohydrates per can, while the food from brand B contains 20 g of protein and 150 g of carbohydrates per can. If the ideal meal consists of 15 g of protein and 145 g of carbohydrates, how many cans of each brand should be used?